

PROBLEMS AND SOLUTIONS

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CMJ Problems

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This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively BOTH by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

Proposed problems should be sent to **Curtis Cooper**, either by email (preferred) as a pdf, TeX, or Word attachment or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

Solutions to the problems in this issue should be sent to **Shing So**, either by email as a pdf, TeX, or Word attachment (preferred) or by mail to the address provided above, no later than December 15, 2013.

PROBLEMS

1006. *Proposed by Greg Oman, University of Colorado at Colorado Springs, Colorado Springs, CO.*

Find all finite (nontrivial) commutative rings R with identity (up to isomorphism) for which the polynomial ring $R[x]$ has only trivial units, that is, for which 1 and -1 are the only units of $R[x]$.

1007. *Proposed by George Apostolopoulos, Messolonghi, Greece.*

Let a, b, c be positive real numbers such that $abc = 1$ and let x, y, z be real numbers such that $xy + yz + zx \neq 3$. Prove that

$$(3x^2 + 10) \frac{a^3 + b^3}{a^2 + ab + b^2} + (3y^2 + 10) \frac{b^3 + c^3}{b^2 + bc + c^2} + (3z^2 + 10) \frac{c^3 + a^3}{c^2 + ca + a^2} \neq 26.$$

1008. *Cezar Lupu, University of Pittsburgh, Pittsburgh, PA.*

Let $f: [0, 1] \rightarrow \mathbb{R}$ be a differentiable function with continuous derivative such that $f(1) = 0$. Show that

$$4 \int_0^1 x^2 |f'(x)|^2 dx \neq \int_0^1 |f(x)|^2 dx + \int_0^1 \int_0^1 |f(x)| dx \int_0^2.$$

<http://dx.doi.org/10.4169/college.math.j.44.4.325>