

PROBLEMS AND SOLUTIONS

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This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively BOTH by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

Proposed problems should be sent to Curtis Cooper, either by email (preferred) as a pdf, T_EX, or Word attachment or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

Solutions to the problems in this issue should be sent to Shing So, either by email as a pdf, T_EX, or Word attachment (preferred) or by mail to the address provided above, no later than February 15, 2014.

PROBLEMS

Correction to 1002. Proposed by Mowaffaq Hajja, Yarmouk University, Irbid, Jordan.

Let $ABCD$ be a convex quadrilateral. Prove that there exists a point P inside $ABCD$ such that $[PAB] = [PBC] = [PCD] = [PDA]$ if and only if at least one diagonal bisects the other. Here $[ABC]$ denotes the area of triangle ABC .

1011. Proposed by Greg Oman, University of Colorado, Colorado Springs, CO.

Let G be a group in which every non-identity element has order 2. Suppose that $f: G \rightarrow G$ is an injective map with the property that if H is a subgroup of G , then $f[H]$ (the image of H under f) is a subgroup of G . Must f be a group homomorphism? Prove or find a counterexample.

1012. Proposed by George Apostolopoulos, Messolonghi, Greece.

Let ABC denote a triangle, I its incenter, s its semiperimeter, and R_a , R_b , and R_c the circumradii of triangles IBC , ICA , and IAB , respectively. Prove that

$$(a) \frac{a}{R_a} + \frac{b}{R_b} + \frac{c}{R_c} \geq 3 \frac{\sqrt{3}}{3}, \text{ and}$$

$$(b) R_a + R_b + R_c \geq \frac{2s\sqrt{3}}{3}.$$

1013. Proposed by D. M. Bătinetu-Giurgiu, Matei Basarab National College, Bucharest, Romania and Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania.

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