**1054.** *Proposed by Greg Oman, University of Colorado, Colorado Springs, CO.*

Let $R$ be the set of all infinite integer-valued sequences $(x_n)$ which are ultimately constant (that is, functions $f : \mathbb{N} \to \mathbb{Z}$ such that there is an integer $N$ with $x_m = x_n$ for all $m, n \geq N$). Also, let $S$ be the set of all bi-infinite integer-valued sequences $(x_n)$ which are ultimately constant on both the left and right (that is, functions $f : \mathbb{Z} \to \mathbb{Z}$ such that there are integers $M$ and $N$ with $x_m = x_n$ for all $m, n \leq M$ and for all $m, n \geq N$). These $R$ and $S$ are rings under coordinatewise addition and multiplication. Are $R$ and $S$ isomorphic rings?

**1055.** *Proposed by D. M. Bătinețu-Giurgiu, Matei Basarab National College, Bucharest, Romania and Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania.*

Given a triangle $ABC$, let $R$ be its circumradius and $r$ its inradius. Draw three lines which are tangent to the incircle of $ABC$ and parallel to the sides of the triangle. In this way, three triangles inside triangle $ABC$ are formed. Let $r_1$, $r_2$, $r_3$ be the radii of the inscribed circles of the three triangles. Prove

(a) $r_1 + r_2 + r_3 = r$ and

(b) $\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} = \dfrac{r + 4R}{r^2}$.

## SOLUTIONS

### Another algebraic inequality

**1026.** *Proposed by Elias Lampakis, Kiparissia, Greece.*

Let $a, b, c$ be positive real numbers. Prove that

$$\frac{2a^2 - ab - b^2}{a + b} + \frac{2b^2 - bc - c^2}{b + c} + \frac{2c^2 - ca - a^2}{c + a} \geq \frac{a^2 + b^2 + c^2 - ab - bc - ca}{a + b + c}.$$

*Solution by Tom Jager, Calvin College, Grand Rapids, MI.*

Since $2x^2 - xy - y^2 = \frac{1}{2}(x - y)^2 + \frac{3}{2}(x^2 - y^2)$ and $a, b, c$ are positive,

$$\begin{aligned}
&\frac{2a^2 - ab - b^2}{a + b} + \frac{2b^2 - bc - c^2}{b + c} + \frac{2c^2 - ca - a^2}{c + a} \\
&= \frac{1}{2}\left[\frac{(a-b)^2}{a+b} + \frac{(b-c)^2}{b+c} + \frac{(c-a)^2}{c+a}\right] + \frac{3}{2}\left[\frac{a^2 - b^2}{a+b} + \frac{b^2 - c^2}{b+c} + \frac{c^2 - a^2}{c+a}\right] \\
&= \frac{1}{2}\left[\frac{(a-b)^2}{a+b} + \frac{(b-c)^2}{b+c} + \frac{(c-a)^2}{c+a}\right] \\
&\geq \frac{1}{2} \cdot \frac{(a-b)^2 + (b-c)^2 + (c-a)^2}{a+b+c} \\
&= \frac{a^2 + b^2 + c^2 - ab - bc - ca}{a+b+c}.
\end{aligned}$$

*Also solved by* ADNAN ALI, Atomic Energy Central School-4, Mumbai, India; ARKADY ALT, San Jose, CA; MICHEL BATAILLE, Rouen, France; BRIAN BRADIE, Christopher Newport U.; DIONNE BAILEY, ELSIE CAMPBELL, AND CHARLES DIMINNIE (jointly), Angelo State U.; SUBRAMANYAM DURBHA, C. C. of Philadelphia;