

# PROBLEMS AND SOLUTIONS

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This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively *both* by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

**Proposed problems** should be sent to **Curtis Cooper**, either by email (preferred) as a pdf, T<sub>E</sub>X, or Word attachment or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

**Solutions to the problems in this issue** should be sent to **Chip Curtis**, either by email as a pdf, T<sub>E</sub>X, or Word attachment (preferred) or by mail to the address provided above, no later than May 15, 2016.

## PROBLEMS

**1066.** *Proposed by George Apostolopoulos, Messolonghi, Greece.*

Let  $a, b, c$  be the lengths of the sides of a triangle  $ABC$  with inradius  $r$  and circumradius  $R$ . Prove that

$$(a + b) \tan \frac{C}{2} + (b + c) \tan \frac{A}{2} + (c + a) \tan \frac{B}{2} = 4(R + r).$$

**1067.** *Proposed by Greg Oman, University of Colorado, Colorado Springs, CO.*

Find all (nontrivial) commutative Artinian rings  $R$  for which 1 and  $-1$  are the only units of  $R$ .

**1068.** *Proposed by Spiros P. Andriopoulos, Third High School of Amaliada, Eleia, Greece.*

Let  $f_1(x), f_2(x), \dots, f_n(x)$  be continuous and positive on  $[0, 1]$ . Prove that

$$\int_0^1 \frac{f_1(x)}{f_2(1-x)} dx \cdot \int_0^1 \frac{f_2(x)}{f_3(1-x)} dx \cdots \int_0^1 \frac{f_n(x)}{f_1(1-x)} dx \geq 1.$$

**1069.** *Proposed by Ángel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

Let  $\{u_n\}_{n \geq 0}$  be a sequence defined recursively by  $u_0 \geq 0$ ,  $u_1 \geq 0$ , and  $u_{n+1} = \sqrt{u_n \cdot u_{n-1}}$ , for  $n \geq 1$ . Determine  $\lim_{n \rightarrow \infty} u_n$  in terms of  $u_0, u_1$ .

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<http://dx.doi.org/10.4169/college.math.j.47.1.61>