2063. *Proposed by Ovidiu Furdui and Alina Sîntămărian, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.*

Evaluate

$$\sum_{n=0}^{\infty}\sum_{k=1}^{\infty}\frac{(-1)^{n+k-1}}{(n+k)^2}.$$

2064. *Proposed by Ioan Băetu, Botoşani, Romania.*

Characterize those integers $n \geq 2$ such that the ring $\mathbb{Z}_n$ of integers modulo n has a subset F that is a field under the operations of addition and multiplication induced from $\mathbb{Z}_n$. [Note that the unity i of such a field F need not be the unity 1 of $\mathbb{Z}_n$.]

2065. *Proposed by Su Pernu Mero, Valenciana GTO, Mexico.*

Let $\mathcal{Q}$ be a cube centered at the origin of $\mathbb{R}^3$. Choose a unit vector (a, b, c) uniformly at random on the surface of the unit sphere $a^2 + b^2 + c^2 = 1$, and let Π be the plane $ax + by + cz = 0$ through the origin and normal to (a, b, c). What is the probability that the intersection of Π with $\mathcal{Q}$ is a hexagon?

Quickies

1087. *Proposed by Michel Bataille, Rouen, France.*

Let $H_0 = 0$ and $H_n = \sum_{k=1}^{n} k^{-1}$ for $n \in \mathbb{N}$. Given positive integers m, n, prove that

$$\sum_{k=1}^{n} k(H_{m+k} - H_{k-1}) + \sum_{k=1}^{m} k(H_{n+k} - H_{k-1}) = mn + m + n.$$

1088. *Proposed by Luke Harmon and Greg Oman, University of Colorado, Colorado Springs, CO.*

A binary operation $*$ on a set S is *injective* if, for all $a, b, c, d \in S$, the equality $a * b = c * d$ implies $a = c$ and $b = d$. Is there an infinite set with an associative and injective binary operation?

Solutions

Nonarchimedean convexity and an integral inequality **February 2018**

2036. *Proposed by Dan Stefan Marinescu, Hunedoara City and Leonard Giugiuc, Drobeta Turnu-Severin, Romania.*

Let a and b be real numbers with $a < b$. Let $f : [a, b] \to \mathbb{R}$ be a continuous function such that $f(tx + (1 - t)y) \leq \max\{f(x), f(y)\}$ for all $x, y \in [a, b]$ and $t \in [0, 1]$. Prove that if $f(a) = 0$ and $\int_a^b f(x)\,dx = 0$ then $\int_a^b f(x)g(x)\,dx \geq 0$ for all increasing functions $g : [a, b] \to \mathbb{R}$.

Solution by Tom Jager, Calvin College, Grand Rapids, MI.
Let $L = \sup\{x \in [a, b] : f(x) \leq 0\}$. (Since $f(a) = 0$, the supremum above does exist.) We have $f(L) \leq 0$ by continuity of f; thus, for $x \in [a, L]$ we have $f(x)$