

# PROBLEMS AND SOLUTIONS

## EDITORS

**Curtis Cooper**  
CMJ Problems  
Department of Mathematics and  
Computer Science  
University of Central Missouri  
Warrensburg, MO 64093  
cmjproblems@maa.org

**Charles N. Curtis**  
CMJ Solutions  
Mathematics Department  
Missouri Southern State University  
3950 Newman Road  
Joplin, MO 64801  
cmjsolutions@maa.org

This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively *both* by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

**Proposed problems** should be sent to **Curtis Cooper**, either by email (preferred) as a pdf,  $\text{\TeX}$ , or Word attachment or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

**Solutions to the problems in this issue** should be sent to **Chip Curtis**, either by email as a pdf,  $\text{\TeX}$ , or Word attachment (preferred) or by mail to the address provided above, no later than June 15, 2017.

## PROBLEMS

**1096.** *Proposed by Greg Oman, University of Colorado, Colorado Springs, CO and Adam Salminen, University of Evansville, Evansville, IN.*

For a commutative ring  $R$  with identity, let  $U(R)$  denote the set of units of  $R$ . Find all finite commutative rings  $R$  with identity for which the set  $U(R) \cup \{0\}$  is closed under addition.

**1097.** *Proposed by Ovidiu Furdui, Mircea Ivan, and Alina Sîntămărian, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.*

Calculate

$$\lim_{a \rightarrow \infty} \frac{1}{a} \int_0^a \frac{x}{1 + a^2 \cos^2 x} dx.$$

**1098.** *Proposed by Michel Bataille, Rouen, France.*

Let  $n$  be a positive integer and  $x_1, x_2, \dots, x_n$  real numbers. Prove that

$$\left( \sum_{k=1}^n \cosh 2x_k \right) \left( \sum_{k=1}^n (\sinh 2x_k)(\tanh 2x_k) \right) \leq 4 \left( \sum_{k=1}^n \cosh^2 x_k \right) \left( \sum_{k=1}^n \sinh^2 x_k \right).$$

---

<http://dx.doi.org/10.4169/college.math.j.48.2.138>