1103. *Proposed by Greg Oman, University of Colorado, Colorado Springs, CO.*
Let $R = \mathbb{Z}[X_i \mid i \in \mathbb{R}]$ be the polynomial ring over $\mathbb{Z}$ in uncountably many variables indexed by the real numbers. Prove or disprove: There exists a countable collection $\{I_n \mid n \in \mathbb{N}\}$ of ideals of R with the following two properties.

(1) The factor ring R/I_n is countable for every $n \in \mathbb{N}$, and

(2) $\bigcap_{n\in\mathbb{N}} I_n = \{0\}$.

Hint: Does there exist a commutative ring S with identity containing R as a subring and a collection $\{I_n \mid n \in \mathbb{N}\}$ of ideals of S which satisfies (1) (with R replaced with S) and (2)?

1104. *Proposed by Greg Oman, University of Colorado, Colorado Springs, CO.*
An *ordered field* consists of a field F along with a binary relation $<$ on F which satisfies the following.

(a) (transitivity) For any $a, b, c \in F$, if $a < b$ and $b < c$, then $a < c$.

(b) (trichotomy) For any $a, b \in F$, exactly one of $a < b$, $a = b$, and $b < a$ holds.

(c) For all $a, b, c \in F$, if $a < b$, then $a + c < b + c$.

(d) For all $a, b, c \in F$, if $a < b$ and $0 < c$, then $ac < bc$.

Now consider dropping the transitivity axiom; call an order $<$ on a field F which satisfies (b), (c), and (d) a *pseudo-order*. Let p be a prime. Show that there exists a pseudo-order on the finite field $\mathbb{Z}/(p)$ if and only if $p \equiv 3 \pmod 4$.

1105. *Proposed by Ángel Plaza, University of Las Palmas de Gran Canaria, Spain.*
Let x, y, z be positive real numbers and k a nonnegative integer. Prove that

$$\sum_{cyclic} \frac{x^{2k+2} + y^{2k+2}}{z^{2k+1}} \geq (xyz)^{k+1} \sum_{cyclic} \frac{1}{x^{3k+2}} + 3\sqrt[3]{xyz}.$$

SOLUTIONS

An inequality for the sides and circumradius of a triangle

1076. *Proposed by D. M. Bătinețu-Giurgiu, Matei Basaraba National College, Bucharest, Romania; and Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania.*

Consider a triangle with sides a, b, and c and circumradius R and $x, y, z > 0$. Prove that

$$\frac{x+y}{za^4} + \frac{y+z}{xb^4} + \frac{z+x}{yc^4} \geq \frac{2}{3R^4}.$$

Solution by George Apostolopoulos, Messolonghi, Greece.

It is well known that for $x, y, z > 0$ we have $(x+y)(y+z)(z+x) \geq 8xyz$. By the AM-GM inequality,

$$\frac{x+y}{za^4} + \frac{y+z}{xb^4} + \frac{z+x}{yc^4} \geq 3\sqrt[3]{\frac{(x+y)(y+z)(z+x)}{xyz(abc)^4}}$$

$$\geq 3\sqrt[3]{\frac{8xyz}{xyz(abc)^4}} = \frac{6}{(\sqrt[3]{abc})^4}.$$