PROBLEMS AND SOLUTIONS

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Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the inside front cover. Submitted solutions should arrive at that address before December 31, 2005. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11166. *Proposed by Greg Oman, The Ohio State University, Columbus, OH.* Let R_C denote the ring of continuous functions $f\colon \mathbb{R} \to \mathbb{R}$, and let R_D denote the subring of R_C consisting of those elements of R_C that are differentiable on $\mathbb{R}$. Are the rings R_C and R_D isomorphic?

11167. *Proposed by Vicenţiu Rădulescu, University of Craiova, Craiova, Romania.* Let Ω be the set of all complex numbers z satisfying $0 < |z| < 1$. Fix a positive integer n, and for arbitrary distinct elements $z_1, \dots, z_n$ of Ω, define

$$\begin{aligned} f(z_1, \dots, z_n) = {} & \prod_{j=1}^{n} |z_j|^2(1 - |z_j|^2) \cdot \prod_{1\le j<k\le n} |z_j| \cdot |z_k| \cdot |z_j - z_k|^2 \cdot \\ & \prod_{1\le j<k\le n} |z_j| \cdot |z_k| \cdot \left[|z_j - z_k|^2 + (1 - |z_j|^2)(1 - |z_k|^2)\right]. \end{aligned}$$

(a) For $n = 2$ prove that the maximum of f is attained by a unique configuration (up to a rotation) that consists of two points symmetric with respect to the origin.
(b) For $n = 3$ prove that the maximal configuration for f is also unique (up to a rotation) and consists of three points that are the vertices of an equilateral triangle centered at the origin.

11168. *Proposed by Kent Holing, Trondheim, Norway.* Let a, b, and c be the sides of a Pythagorean triangle, with c the hypotenuse.
(a) Show that $c - a$ and $c + a$ cannot both be sides of a single Pythagorean triangle.
(b) Show that none of $c^2 + 4ab$, $c^2 - 4ab$, or $c^2 - 9a^2$ can be square.

11169. *Proposed by Mohammad Hossein Mehrabi, University of Science and Technology, Tehran, Iran.* Let ϕ be the function on the positive real numbers given by $\phi(x) = (e/x)^x\Gamma(x)$, where Γ is the unique log convex function on the positive real numbers satisfying $\Gamma(n) = (n-1)!$ for positive integers n. Prove that ϕ is strictly decreasing on $(0, \infty)$.