PROBLEMS AND SOLUTIONS

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Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the inside front cover. Submitted solutions should arrive at that address before July 31, 2007. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11284. *Proposed by Greg Oman, Ohio State University, Columbus, OH.* Let R be an infinite commutative ring with identity. Suppose that every proper ideal of R has smaller cardinality than R. Prove that R is a field.

11285. *Proposed by Yakub Aliyev, Qafqaz University and Baku State University, Baku, Azerbaijan.* Let six points be chosen in cyclic order on the sides of triangle ABC: A_1 and A_2 on BC, B_1 and B_2 on CA, and C_1 and C_2 on AB. Let K denote the intersection of A_1B_2 and C_1A_2, L the intersection of B_1C_2 and A_1B_2, and M the intersection of C_1A_2 and B_1C_2. Let T, U, and V be the intersections of A_1B_2 and B_1A_2, B_1C_2 and B_2C_1, and C_1A_2 and C_2A_1, respectively. Prove that lines AK, BL, and CM are concurrent if and only if points T, U, and V are collinear.

11286. *Proposed by Marc LeBrun, Fixpoint Inc., Larkspur, CA, and David Applegate and N.J.A. Sloane, AT&T Shannon Labs, Florham Park, NJ.* When a and b are positive integers with $b \geq 10$, write a_b (or $a : b$ inline) for the integer whose base b expansion is the decimal expansion of a. That is, if $a = \sum_{i=0}^{k} a_i 10^i$ with each a_i in $\{0, 1, \dots, 9\}$, then $a_b = a : b = \sum_{i=0}^{k} a_i b^i$. Thus,

$$10_{11_{12_{13}}} = 10 : (11 : (12 : 13)) = 16.$$

Consider the "dungeon sequences"

$$10,\ 10:11,\ (10:11):12,\ ((10:11):12):13\dots,$$

$$10,\ 10:11,\ 10:(11:12),\ 10:(11:(12:13))\dots,$$

$$10,\ 11:10,\ (12:11):10,\ ((13:12):11):10\dots,$$

$$10,\ 11:10,\ 12:(11:10),\ 13:(12:(11:10))\dots.$$

Let s_n be the nth term in any of these sequences. Show that $\log\log s_n/(n \log\log n)$ approaches 1 as n goes to infinity.