$$\limsup_{n\to\infty} \frac{\log |A_{m,n}|}{\log n} = 1.$$

1144. *Proposed by Andrew Wu, St. Albans School, McLean, VA.*

Let ABC be an acute, scalene triangle with circumcenter O, and let D be a point lying on side $\overline{BC}$. The perpendicular bisectors of $\overline{DB}$ and $\overline{DC}$ meet lines $\overline{AB}$ and $\overline{AC}$, say at points P and Q, respectively. Let X be the reflection of D about $\overline{PQ}$. Show that $\overline{AX} \parallel \overline{BC}$ if and only if D, O, and X are collinear.

1145. *Proposed by Greg Oman, University of Colorado at Colorado Springs, Colorado Springs, CO.*

Let $\{G_i : i \in I\}$ be a collection of groups, where I is some nonempty index set. Then the direct product of the G_i, denoted $\prod_{i\in I} G_i$, is the group whose elements are sequences $(g_i : i \in I)$, where $g_i \in G_i$ for each i. Multiplication is componentwise: $(g_i : i \in I)(h_i : i \in I) := (g_i h_i : i \in I)$. In case $G_i = G$ for all i, then we agree to denote $\prod_{i\in I} G_i$ by $\prod_I G$.

(a) Does there exist a nontrivial group G such that for all groups H and nonempty *finite* index sets I: $\prod_I G \cong \prod_I H$ implies $G \cong H$? Give an example of such a G or prove that no such group exists.

(b) Does there exist a nontrivial group G such that for all groups H and *infinite* index sets I: $\prod_I G \cong \prod_I H$ implies $G \cong H$? Give an example of such a G or prove that no such group exists.

SOLUTIONS

An asymmetric inequality in three variables

1116. *Proposed by Mehtaab Sawhney (student), Massachusetts Institute of Technology, Cambridge, MA.*

Prove that

$$\sum_{\text{cyc}} x^4 + \sum_{\text{cyc}} \left(\sqrt{3} - 1\right) x^2yz \geq \sum_{\text{cyc}} x^3y,$$

for all reals x, y, and z.

1. Solution 1 by *Subhankar Gayen, West Bengal, India*: Set

$$F(x, y, z) = \sum_{\text{cyc}} \left(x^2 - 2y^2 + z^2 + kyz - kzx\right)^2.$$

For $k = \sqrt{3}$, the claimed inequality is equivalent to $F(x, y, z) \geq 0$.

2. Solution 2 by *Ioannis Sfikas, National and Kapodistrian University of Athens, Greece*: The claimed inequality is equivalent to

$$\sum \text{cyc} \left[\sqrt{3}\left(x^2 - y^2\right) - xy + 2yz - zx\right]^2 \geq 0.$$

Also solved by the proposer. Sfikas also provided a second solution.