1149. *Proposed by George Stoica, St. John, New Brunswick, Canada.*

Prove that the following equation holds for any real $x > 1$ and natural number $n \geq 2$:

$$\frac{1}{n}\sum_{i=0}^{n-1}\binom{n-1}{i}^{-1}\left(\sum_{j=0}^{i}\binom{n}{j}(x-1)^{i-j}\right)=\frac{1}{x}\sum_{k=1}^{n}\frac{x^k}{k}.$$

1150. *Proposed by Greg Oman, University of Colorado at Colorado Springs, Colorado Springs, CO.*

Let R be the set of all infinite convergent sequences of real numbers, and let S be the set of all infinite sequences of real numbers, convergent or not. Then S becomes a ring via componentwise addition and multiplication of sequences. Further, R is a subring of S. Are R and S isomorphic as rings? Prove or disprove.

SOLUTIONS

A trigonometric sum

1121. *Proposed by George Stoica, Saint John, New Bruswick.*

Prove that

$$\sum_{j=1}^{n}(-1)^{j-1}\cos^{2k}\frac{j\pi}{2n+2}=\frac{1}{2}$$

for all $k = 1, \ldots, n$.

Solution by William Seaman, Bethlehem, PA.

The given sum is successively equal to

$$\left(\frac{1}{2}\right)^{2k}\sum_{j=1}^{n}(-1)^{j-1}\left(e^{\frac{j\pi}{2n+2}i}+e^{-\frac{j\pi}{2n+2}i}\right)^{2k}$$

$$=\left(\frac{1}{2}\right)^{2k}\sum_{j=1}^{n}(-1)^{j-1}\sum_{p=0}^{2k}\binom{2k}{p}e^{\frac{pj\pi}{2n+2}i}e^{-\frac{(2k-p)j\pi}{2n+2}i}$$

$$=\left(\frac{1}{2}\right)^{2k}\sum_{j=1}^{n}(-1)^{j-1}\sum_{p=0}^{2k}\binom{2k}{p}e^{\frac{(p-k)j\pi}{n+1}i}$$

$$=-\left(\frac{1}{2}\right)^{2k}\sum_{p=0}^{2k}\binom{2k}{p}\sum_{j=1}^{n}\left(-e^{\frac{(p-k)\pi}{n+1}i}\right)^{j}.$$

The second sum is a geometric series, and the above is thus equal to

$$-\left(\frac{1}{2}\right)^{2k}\sum_{p=0}^{2k}\binom{2k}{p}\left[\frac{1-\left(-e^{\frac{(p-k)\pi}{n+1}i}\right)^{n+1}}{1-\left(-e^{\frac{(p-k)\pi}{n+1}i}\right)}-1\right].$$

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