PROBLEMS AND SOLUTIONS

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Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the back of the title page. Proposed problems should never be under submission concurrently to more than one journal. Submitted solutions should arrive before May 31, 2012. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11614. *Proposed by Moubinool Omarjee, Lycée Jean-Lurçat, Paris, France.* Let α be a real number with $\alpha > 1$, and let $\{u_n\}_{n\in\mathbb{N}}$ be a sequence of positive numbers such that $\lim_{n\to\infty} u_n = 0$ and $\lim_{n\to\infty}(u_n - u_{n+1})/u_n^\alpha$ exists and is nonzero. Prove that $\sum_{n=1}^{\infty} u_n$ converges if and only if $\alpha < 2$.

11615. *Proposed by Constantin Mateescu, Zinca Golescu High School, Pitesti, Romania.* Let A, B, and C be the vertices of a triangle, and let K be a point in the plane distinct from these vertices and the lines connecting them. Let M, N, and P be the midpoints of BC, CA, and AB, respectively. Let D, E, and F be the intersections of the lines through MK and NP, NK and PM, and PK and MN, respectively. Prove that the parallels from D, E, and F to AK, BK, and CK, respectively, are concurrent.

11616. *Proposed by Stefano Siboni, University of Trento, Trento, Italy.* Let $x_1, \dots, x_n$ be distinct points in $\mathbb{R}^3$, and let $k_1, \dots, k_n$ be positive real numbers. A test object at x is attracted to each of $x_1, \dots, x_n$ with a force along the line from x to x_j of magnitude $k_j\|x - x_j\|^2$, where $\|u\|$ denotes the usual euclidean norm of u. Show that when $n \geq 2$ there is a unique point x^* at which the net force on the test object is zero.

11617. *Proposed by Greg Oman, University of Colorado at Colorado Springs, Colorado Springs, CO.* Let C be the ring of continuous functions on $\mathbb{R}$, equipped with pointwise addition and pointwise multiplication. Let D be the ring of differentiable functions on $\mathbb{R}$, equipped with the same addition and multiplication. The ring identity in both cases is the function f_1 on $\mathbb{R}$ that sends every real number to 1. Is there a subring E of D, containing f_1, that is isomorphic to C? (The ring isomorphism must carry f_1 to f_1.)

11618. *Proposed by Pál Péter Dályay, Szeged, Hungary.* Let a, b, c, and d be real numbers such that $a < c < d < b$ and $b - a = 2(d - c)$. Let S be the set of twice-differentiable functions from $[a, b]$ to $\mathbb{R}$ with continuous second derivative such that

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