PROBLEMS AND SOLUTIONS

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Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the back of the title page. Proposed problems should never be under submission concurrently to more than one journal. Submitted solutions should arrive before December 31, 2012. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11656. *Proposed by Valerio De Angelis, Xavier University of Louisiana, New Orleans, LA.* The *sign chart* of a polynomial f with real coefficients is the list of successive pairs (ϵ, σ) of signs of (f', f) on the intervals separating real zeros of ff', together with the signs at the zeros of ff' themselves, read from left to right. Thus, for $f = x^3 - 3x^2$, the sign chart is $((1,-1),(0,0),(-1,-1),(0,-1),(1,-1),(1,0),(1,1))$. As a function of n, how many distinct sign charts occur for polynomials of degree n?

11657. *Proposed by Gregory Galperin, Eastern Illinois University, Charleston, IL, and Yury Ionin, Central Michigan University, Mount Pleasant, MI.* Given a set V of n points in $\mathbb{R}^2$, no three of them collinear, let E be the set of $\binom{n}{2}$ line segments joining distinct elements of V.
(a) Prove that if $n \not\equiv 2 \pmod 3$, then E can be partitioned into triples in which the length of each segment is greater than the sum of the other two.
(b) Prove that if $n \equiv 2 \pmod 3$ and e is an element of E, then $E\backslash\{e\}$ can be so partitioned.

11658. *Proposed by Greg Oman, University of Colorado at Colorado Springs, Colorado Springs, CO.* Let V be the vector space over $\mathbb{R}$ of all (countably infinite) sequences $(x_1, x_2, \dots)$ of real numbers, equipped with the usual addition and scalar multiplication. For $v \in V$, say that v is *binary* if $v_k \in \{0, 1\}$ for $k \geq 1$, and let B be the set of all binary members of V. Prove that there exists a subset I of B with cardinality $2^{\aleph_0}$ that is linearly independent over $\mathbb{R}$. (An infinite subset of a vector space is linearly independent if all of its finite subsets are linearly independent.)

11659. *Proposed by Albert Stadler, Herrliberg, Switzerland.* Let x be real with $0 < x < 1$, and consider the sequence $\langle a_n \rangle$ given by $a_0 = 0$, $a_1 = 1$, and, for $n > 1$,

$$a_n = \frac{a_{n-1}^2}{xa_{n-2} + (1-x)a_{n-1}}.$$

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