PROBLEMS AND SOLUTIONS

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This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively *both* by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

Proposed problems should be sent to **Greg Oman**, either by email (preferred) as a pdf, TEX, or Word attachment or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

Solutions to the problems in this issue should be sent to **Chip Curtis**, either by email as a pdf, TEX, or Word attachment (preferred) or by mail to the address provided above, no later than July 15, 2020. Sending both pdf and TEXfiles is ideal.

PROBLEMS

1166. *Proposed by Greg Oman, University of Colorado, Colorado Springs, Colorado Springs, CO.*

Let R be a nontrivial left Noetherian ring, not assumed to have an identity (recall that R is *left Noetherian* if there does not exist an infinite, strictly ascending chain $I_0 \subsetneq I_2 \subsetneq \cdots$ of left ideals of R). Prove the following: for any $r, s \in R$, there exist $x, y \in R$, not both zero, such that $xr = ys$.

1167. *Proposed by Paul Bracken, University of Texas Rio Grande Valley, Edinburg, TX.*

Prove that the following equations hold:

1. $\sum_{n=0}^{\infty} \binom{2n}{n} \frac{1}{4^n} \frac{1}{n+1} = 2,$
2. $\sum_{n=0}^{\infty} \binom{2n}{n} \frac{1}{4^n (n+1)^2} = 4 - 4\ln(2)$, and
3. $\sum_{n=0}^{\infty} \binom{2n}{n} \frac{1}{4^n (n+1)^3} = 4(\ln(2))^2 - 8\ln(2) - \frac{\pi^2}{3} + 8.$

1168. *Proposed by George Stoica, St. John's University, New Brunswick, Canada.*

Let $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ be unit vectors in a Hilbert space with dot product $(\cdot, \cdot)$. Prove the following inequality:

$$\text{Re}^2(\mathbf{x}, \mathbf{y}) + \text{Re}^2(\mathbf{y}, \mathbf{z}) + \text{Re}^2(\mathbf{x}, \mathbf{z}) \le 1 + 2\text{Re}(\mathbf{x}, \mathbf{y})\text{Re}(\mathbf{y}, \mathbf{z})\text{Re}(\mathbf{x}, \mathbf{z}),$$

where $\text{Re}(\cdot, \cdot)$ denotes the real part of the dot product $(\cdot, \cdot)$.

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