PROBLEMS AND SOLUTIONS

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Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the back of the title page. Proposed problems should never be under submission concurrently to more than one journal. Submitted solutions should arrive before August 31, 2013. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11698. *Proposed by Timothy Hall, Cambridge, MA.* Provide an algorithm that takes as input a positive integer n and a nonzero constant k and returns polynomials F and G in variables u and v such that when x^n is substituted for u, and $x + k/x$ is substituted for v, $F(u, v)/G(u, v)$ simplifies (disregarding removable singularities) to x. (For example, when $k = 1$ and $n = 3$, $F = u + v$ and $G = v^2 - 1$ will do.)

11699. *Proposed by Bakir Farhi, University of Bejaia, Bejaia, Algeria.* Let (a_k) be a strictly increasing sequence of positive integers such that $\sum_{k=2}^{\infty} \frac{1}{a_k \log a_k}$ diverges. Prove that $\operatorname{lcm}(a_1, \dots, a_k) = \operatorname{lcm}(a_1, \dots, a_{k+1})$ for infinitely many k in $\mathbb{N}$.

11700. *Proposed by Evan O'Dorney (student), Harvard University, Cambridge, MA.* Let n be an integer greater than 1. Let a, b, and c be complex numbers with $a + b + c = a^n + b^n + c^n = 0$. Prove that the absolute values of a, b, and c cannot be distinct.

11701. *Proposed by D. M. Bătinețu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, and Neculai Stanciu, "George Emil Palade" Secondary School, Buzău, Romania.*
(a) Let (x_n) be the sequence defined by $\sum_{k=1}^{mn} 1/k = \phi + \log(mn + x_n)$, where ϕ is the Euler-Mascheroni constant. Find $\lim_{n\to\infty} x_n$.
(b) Let (y_n) be the sequence defined by $\sum_{k=1}^{mn} 1/k = \phi + \log(m(n + y_n))$. Find $\lim_{n\to\infty} y_n$.

11702. *Proposed by Greg Oman, University of Colorado at Colorado Springs, Colorado Springs, CO.* Find all nonzero rings R (not assumed to be commutative or to contain a multiplicative identity) with these properties:
(a) There exists $x \in R$ that is neither a left nor a right zero divisor, and
(b) Every map ϕ from R to R that satisifes $\phi(x + y) = \phi(x) + \phi(y)$ also satisfies $\phi(xy) = \phi(x)\phi(y)$. (That is, every additive homomorphism on R is a ring homomorphism.)

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