

PROBLEMS AND SOLUTIONS

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with the collaboration of Itshak Borosh, Paul Bracken, Ezra A. Brown, Randall Dougherty, Tamás Erdélyi, Zachary Franco, Christian Friesen, Ira M. Gessel, László Lipták, Frederick W. Luttmann, Vania Mascioni, Frank B. Miles, Richard Pfliefer, Dave Renfro, Cecil C. Rousseau, Leonard Smiley, Kenneth Stolarsky, Richard Stong, Walter Stromquist, Daniel Ullman, Charles Vanden Eynden, Sam Vandervelde, and Fuzhen Zhang.

Proposed problems and solutions should be sent in duplicate to the MONTHLY problems address on the back of the title page. Proposed problems should never be under submission concurrently to more than one journal. Submitted solutions should arrive before May 31, 2014. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An asterisk () after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11747. Proposed by Jeffrey C. Lagarias, University of Michigan, Ann Arbor, MI. Determine all $n \in \mathbb{N}$ such that $\lfloor n/k \rfloor$ divides n for $1 \leq k \leq n$. Similarly, determine all $n \in \mathbb{N}$ such that $\lfloor n/k \rfloor$ divides n for $1 \leq k \leq n$.

11748. Proposed by Cezar Lupu, University of Pittsburgh, Pittsburgh, PA, and Tudorel Lupu, Decebal High School, Constanta, Romania. Is there a sequence a_1, a_2, \dots of positive real numbers such that $\prod_{k=1}^n \frac{1}{a_k}$ converges, and $\prod_{k=1}^n a_k < n^n$ for all n ?

11749. Proposed by Branko Ćurgus, Western Washington University, Bellingham, WA. For $x \in \mathbb{C}^n$ and $p > 0$, let $\|x\|_p$ denote the standard p -norm on \mathbb{C}^n . Prove that the function $p \mapsto \|x\|_p$ is a strictly decreasing convex function on $(0, \infty)$ if and only if x is not of the form ce_k , where e_k denotes the vector with 1 in the k th position and 0 elsewhere.

11750. Proposed by Greg Oman, University of Colorado at Colorado Springs, Colorado Springs, CO. Prove or disprove that for every integral domain D and every nonzero d in D , there exist infinitely many irreducible polynomials p in the ring $D[x]$ of polynomials in one variable over D such that $p(0) = d$. (A nonzero, nonunit element f of $D[x]$ is irreducible if g or h is a unit of $D[x]$ whenever $gh = f$.)

11751. Proposed by Carol Kempniak, Aliso Niguel High School, Aliso Viejo, CA, and Bogdan Suceavă, California State University, Fullerton, CA. In a triangle with angles of radian measure A , B , and C , prove that

$$\frac{\csc A + \csc B + \csc C}{2} \geq \frac{1}{\sin B + \sin C} + \frac{1}{\sin C + \sin A} + \frac{1}{\sin A + \sin B},$$

with equality if and only if the triangle is equilateral.

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