

PROBLEMS AND SOLUTIONS

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Proposed problems should be submitted online via www.americanmathematicalmonthly.submittable.com. Proposed solutions to the problems below should be submitted on or before April 30, 2017 at the same link. More detailed instructions are available online. Solutions to problems numbered 11921 or lower should continue to be submitted via email to monthlyproblems@math.tamu.edu. Proposed problems must not be under consideration concurrently to any other journal and must not be posted to the internet before the deadline date for solutions. An asterisk () after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11943. *Proposed by Keith Kearnes, University of Colorado, Boulder, CO, and Greg Oman, University of Colorado, Colorado Springs, CO.* Let X be a set, and let \mathcal{F} be a collection of functions f from X into X . A subset Y of X is *closed* under \mathcal{F} if $f(y) \in Y$ for all $y \in Y$ and f in \mathcal{F} . With the axiom of choice given, prove or disprove: There exists an uncountable collection \mathcal{F} of functions mapping \mathbb{Z}^+ into \mathbb{Z}^+ such that (a) every proper subset of \mathbb{Z}^+ that is closed under \mathcal{F} is finite, and (b) for every $f \in \mathcal{F}$, there is a proper infinite subset Y of \mathbb{Z}^+ that is closed under $\mathcal{F} \setminus \{f\}$.

11944. *Proposed by Yury Ionin, Central Michigan University, Mount Pleasant, MI.* Let n be a positive integer, and let $[n] = \{1, \dots, n\}$. For $i \in [n]$, let A_i, B_i, C_i be disjoint sets such that $A_i \cup B_i \cup C_i = [n] - \{i\}$ and $|A_i| = |B_i|$. Suppose also that

$$|A_i \cap B_j| + |B_i \cap C_j| + |C_i \cap A_j| = |B_i \cap A_j| + |C_i \cap B_j| + |A_i \cap C_j|$$

for $i, j \in [n]$. Prove that $i \in A_j$ if and only if $j \in A_i$ and, likewise, for the B s and C s.

11945. *Proposed by Martin Lukarevski, University "Goce Delcev," Stip, Macedonia.* Let a, b , and c be the lengths of the sides of triangle ABC opposite A, B , and C , respectively, and let w_a, w_b, w_c be the lengths of the corresponding angle bisectors. Prove

$$\frac{a}{w_a} + \frac{b}{w_b} + \frac{c}{w_c} \geq 2\sqrt{3}.$$

11946. *Proposed by Moubinoöl Omarjee, Lycée Henri IV, Paris, France.* Let f be a twice differentiable function from $[0, 1]$ to \mathbb{R} with f'' continuous on $[0, 1]$ and $\int_{1/3}^{2/3} f(x) dx = 0$. Prove

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