PROBLEMS AND SOLUTIONS

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Proposed problems should be submitted online at
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Proposed solutions to the problems below should be submitted by August 31, 2018 via the same link. More detailed instructions are available online. Proposed problems must not be under consideration concurrently at any other journal nor be posted to the internet before the deadline date for solutions. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.

PROBLEMS

12034. *Proposed by Gregory Galperin, Eastern Illinois University, Charleston, IL, and Yury J. Ionin, Central Michigan University, Mount Pleasant, MI.* Let N be any natural number that is not a multiple of 10. Prove that there is a multiple of N each of whose digits in base 10 is 1, 2, 3, 4, or 5.

12035. *Proposed by Dinh Thi Nguyen, Tuy Hòa, Vietnam.* Find the minimum value of

$$(a^2+b^2+c^2)\left(\frac{1}{(3a-b)^2}+\frac{1}{(3b-c)^2}+\frac{1}{(3c-a)^2}\right)$$

as a, b, and c vary over all real numbers with $3a \neq b$, $3b \neq c$, and $3c \neq a$.

12036. *Proposed by Greg Oman, University of Colorado, Colorado Springs, CO.* Two metric spaces (X, d) and (X', d') are said to be *isometric* if there is a bijection $\phi : X \to Y$ such that $d(a, b) = d'(\phi(a), \phi(b))$ for all $a, b \in X$. Let X be an infinite set. Find all metrics d on X such that (X, d) and (X', d') are isometric for every subset X' of X of the same cardinality as X. (Here, d' is the metric induced on X' by d.)

12037. *Proposed by José Manuel Rodríguez Caballero, University of Quebec (UQAM), Montreal, QC, Canada.* For a positive integer n, let S_n be the set of pairs (a, k) of positive integers such that $\sum_{i=0}^{k-1}(a+i) = n$. Prove that the set

$$\left\{ n : \sum_{(a,k)\in S_n} (-1)^{a-k} \neq 0 \right\}$$

is closed under multiplication.

12038. *Proposed by George Apostolopoulos, Messolonghi, Greece.* Let ABC be an acute triangle with sides of length a, b, and c opposite angles A, B, and C, respectively, and with medians of length m_a, m_b, and m_c emanating from A, B, and C, respectively. Prove

$$\frac{m_a^2}{b^2+c^2}+\frac{m_b^2}{c^2+a^2}+\frac{m_c^2}{a^2+b^2} \geq 9\cos A\,\cos B\,\cos C.$$

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