Note: After the Fall 2018 pages went to press, several additional solutions were received. These include #1348 by Kenneth Davenport.

#1356: *Proposed by Greg Oman and Ikko Saito, University of Colorado, Colorado Springs.*
Problem. Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(0) = 0$. For $r \in \mathbb{R}$, say that f is *homomorphic at* r if $f(r + x) = f(r) + f(x)$ for all $x \in \mathbb{R}$. Next, set $\mathcal{H}_f := \{r \in \mathbb{R}: f$ is homomorphic at $r\}$. One can check that $\mathcal{H}_f$ is an additive subgroup of $\mathbb{R}$ (which may be assumed in your solution). For the purposes of this problem, say that a subgroup G of $\mathbb{R}$ is *realizable* if $G = \mathcal{H}_f$ for some continuous $f: \mathbb{R} \to \mathbb{R}$ such that $f(0) = 0$.

(a) Prove that every (additive) cyclic subgroup of $\mathbb{R}$ is realizable.
(b) Find all non-cyclic realizable subgroups of $\mathbb{R}$.

#1357: *Proposed by Ron Evans (UCSD) and Steven J. Miller (Williams).*
Let n be a positive integer. A pin of length n units is dropped randomly onto a large floor ruled with equally spaced parallel lines 1 unit apart. When it lands, the pin can intersect k parallel lines, where k is an integer between 0 and $n + 1$ inclusive. If the center of the pin lands halfway between two adjacent lines, which value of k is most probable?

#1358: *Proposed by Ron Evans (UCSD) and Steven J. Miller (Williams).*
Let n be a positive integer. A pin of length n units is dropped randomly onto a large floor ruled with equally spaced parallel lines 1 unit apart. When it lands, the pin can intersect k parallel lines, where k is an integer between 0 and $n + 1$ inclusive. Which value of k is most probable? (Note unlike the previous problem, now there is no restriction on the location of the center.)

#1359: *Proposed by Robert C. Gebhardt, Chester, NJ.*
Determine the following sums:

$$(a) \quad \frac{1}{1+2} - \frac{1}{3+4} + \frac{1}{5+6} - \frac{1}{7+8} + \cdots$$

$$(b) \quad \frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + \frac{1}{7\cdot 8} + \cdots$$

$$(c) \quad \frac{1}{1\cdot 2} - \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} - \frac{1}{7\cdot 8} + \cdots$$

#1360: *Proposed by Stanley Wu-Wei Liu, East Setauket, Long Island, New York.*
Cutting a cake, be it round or otherwise, is a fun skill with real-world applications. When mathematicians work on such dissection problems starting out with a *quadrilateral*-shaped cake and whimsically demanding that the constituent pieces be *similar polygons* (in the precise Euclidean sense), a lot is known when the number of these similar polygons is chosen to be four. Consider the case of an **isosceles trapezoid with side-length ratios of 1:1:1:2**. There are many fascinating solutions; find at least four partitions of the 1:1:1:2 isosceles trapezoid into four similar polygons.