perpendiculars from I to lines MB and MC, respectively. Prove that the value of

$$\frac{IE+IF}{AM}$$

is independent of the position of M.

1810. *Proposed by Greg Oman, Otterbein College, Westerville, OH.*

Let R be a ring. For elements $x, y \in R$ we say x divides y on the right if and only if there is a $z \in R$ with $xz = y$. (We denote this by $x|_r y$.) An element $p \in R$ is a right prime if and only if whenever $p|_r xy$, then either $p|_r x$ or $p|_r y$. Prove that if every element of R is right prime, then R is a division ring, that is, the nonzero elements of R form a group under multiplication. (Note: R is not assumed to be commutative nor is it assumed that R has a multiplicative identity.)

Quickies

Answers to the Quickies are on page 381.

Q985. *Proposed by Ovidiu Furdui, Campia-Turzii, Cluj, Romania.*

Let x be a real number. Evaluate the sum

$$\sum_{n=1}^{\infty} n^2 \left(e^x - 1 - \frac{x}{1!} - \frac{x^2}{2!} - \cdots - \frac{x^n}{n!}\right).$$

Q986. *Proposed by Peter Ross, Santa Clara University, Santa Clara, CA.*

Prove that in a given ellipse, there exist infinitely many inscribed triangles of maximal area.

Solutions

Growth of $\gamma_n - \gamma$ **December 2007**

1781. *Proposed by Paul Bracken, University of Texas, Edinburg, TX.*

Let γ be Euler's constant and for positive integer n define

$$\gamma_n = \sum_{k=1}^{n} \frac{1}{k} - \log n \qquad \text{and} \qquad \alpha_n = 2n(\gamma_n - \gamma).$$

Prove that the sequence $\{\alpha_n\}$ is monotonically increasing and bounded above. In addition, determine $\lim_{n\to\infty} \alpha_n$.

Solution by Ángel Plaza, University of Las Palmas de Gran Canaria, Las Palmas G.C., Spain.

For $n \geq 1$, define the sequence $\{\beta_n\}$ by $\beta_n = \gamma_n - \gamma - \dfrac{1}{2n}$. Then, $2n\beta_n = \alpha_n - 1$. For $n \geq 1$, we have

$$\begin{aligned}
\beta_{n+1} - \beta_n &= \frac{1}{n+1} - \log\left(1+\frac{1}{n}\right) + \frac{1}{2n(n+1)} \\
&= \frac{1}{n} - \log\left(1+\frac{1}{n}\right) - \frac{1}{2n(n+1)} = f(n),
\end{aligned}$$