

1934. Proposed by Greg Oman, University of Colorado, Colorado Springs, CO.

Let R be an infinite ring (not assumed commutative or to contain an identity). Consider the following two conditions:

- (a) R has zero divisors; that is, there exist nonzero elements $s, t \in R$ such that $st = 0$.
 (b) There exist nonzero elements $s, t \in R$ such that $st = 0$. Further, $Rs \neq \{0\}$ and $Rt \neq \{0\}$ (here, $Rs := \{rs : r \in R\}$).

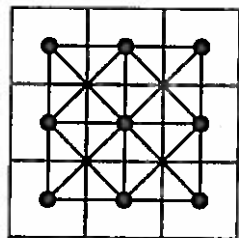
Does (a) imply that R possesses a nonzero left ideal I such that R and R/I have the same cardinality? Does the answer change if we assume (b) instead?

1935. Proposed by Stan Wagon, Macalester College, St. Paul, MN.

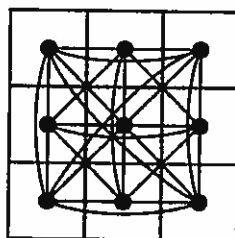
Let $K_{m,n}$ be the graph on the vertex set $\{1, 2, \dots, m\} \times \{1, 2, \dots, n\}$, where the vertex (m_1, n_1) is connected to the vertex (m_2, n_2) if the king piece in chess can move from the square (m_1, n_1) to the square (m_2, n_2) . Define $Q_{m,n}$ accordingly for the possible queen moves on an $m \times n$ chessboard.

- (a) For which pairs (m, n) is $K_{m,n}$ perfect?
 (b) For which pairs (m, n) is $Q_{m,n}$ perfect?

Note: A graph is *perfect* if neither the graph nor its complement has a chordless odd cycle of length 5 or more.



$K_{3,3}$



$Q_{3,3}$

Quickies

Answers to the Quickies are on page 387.

Q1035. Proposed by Jacek Fabrykowski and Thomas Smotzer, Youngstown State University, Youngstown, OH.

Prove that there exists a triangle with sides of lengths a , b , and c if and only if a , b , and c are positive real numbers such that $2a^2b^2 + 2b^2c^2 + 2c^2a^2 > a^4 + b^4 + c^4$.

Q1036. Proposed by Michael W. Botsko, Saint Vincent College, Latrobe, PA.

Let $C = \{f : f \text{ is a continuous real-valued function defined on } \mathbb{R}\}$. In addition, let $D = \{f : f \text{ is a differentiable real-valued function on } \mathbb{R}\}$. Note that both C and D are rings under the usual definitions of addition and multiplication. Finally, let S be any subring of D containing a nonconstant linear function. Prove that C and S are not isomorphic.