PROBLEMS

Proposals

To be considered for publication, solutions should be received by November 1, 2016.

1996. *Proposed by Michael W. Botsko, Saint Vincent College, Latrobe, PA.*

Compute

$$\lim_{x\to 0}\frac{1}{x}\int_0^x \left|\cos\frac{1}{t}\right| dt.$$

1997. *Proposed by Ovidiu Furdui and Alina Sîntămărian, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.*

Calculate

$$\int_0^\infty \left(\frac{1-e^{-x}}{x}\right)^2 dx.$$

1998. *Proposed by Greg Oman, University of Colorado, Colorado Springs, CO.*

Let $\mathbb{N}$ be the set of natural numbers. We call a collection C of subsets of $\mathbb{N}$ *plenary* if there exists a function $f:\mathbb{N}\to\mathbb{N}$ such that $f[X]=\mathbb{N}$ for all $X\in C$, where $f[X] = \{f(x) : x\in X\}$ is the set of images of elements of X under f.

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We invite readers to submit problems believed to be new and appealing to students and teachers of advanced undergraduate mathematics. Proposals must, in general, be accompanied by solutions and by any bibliographical information that will assist the editors and referees. A problem submitted as a Quickie should have an unexpected, succinct solution. Submitted problems should not be under consideration for publication elsewhere.

Proposals and solutions should be written in a style appropriate for this MAGAZINE.

Effective immediately, authors of proposals and solutions should send their contributions using the Magazine's submissions system hosted at `http://mathematicsmagazine.submittable.com`. *More detailed instructions are available there. We hope that this online system will help streamline our editorial team's workflow while still proving accessible and convenient to longtime readers and contributors. We encourage submissions in PDF format, ideally accompanied by LaTeX source. General inquiries to the editors should be sent to* `mathmagproblems@maa.org`.

(a) Prove that any countable collection of infinite subsets of $\mathbb{N}$ is plenary.

(b) Prove that the collection of all infinite subsets of $\mathbb{N}$ is not plenary.

(c) Are there any uncountable plenary collections?

1999. *Proposed by Mihàly Bencze, Brasov, Romania.*

For any real number $a > 1$, evaluate

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m^2 n}{a^m(na^m + ma^n)}.$$

2000. *Proposed by Michel Bataille, Rouen, France.*

Let $\triangle ABC$ have a right angle at A. Let M be the midpoint of AB, let D lie on side $\overline{BC}$ so $BD = BA$, and let P lie on the circumcircle of $\triangle ADC$ so that $\angle APB = 90^\circ$. Let U lie on line $\overleftrightarrow{AP}$ so that $\overline{BU}$ is perpendicular to $\overline{MP}$, and let V lie on $\overleftrightarrow{DP}$ so that $\overline{BV}$ is parallel to $\overline{MP}$.

Prove that $PU/PV = BU/BV$ and the line $\overleftrightarrow{CP}$ bisects $\overline{UV}$.

Quickies

1061. *Proposed by Donald E. Knuth, Stanford University, Stanford, CA.*

If $x^2 = 1 + zx^3$ and $y^2 = 1 - zy^3$ with $x, y > 0$, prove that $xy = 1 + z^2x^3y^3$.

1062. *Proposed by Julien Sorel, Piatra Neamt, Romania.*

Prove that

$$\int_{\sqrt{3}/3}^{\sqrt{3}} \frac{dt}{(1+t^2)^2} = \frac{\pi}{12}$$

without finding the antiderivative of the integrand.

Solutions

An application of Brahmagupta's identity **April 2015**

1966. *Proposed by H. A. ShahAli, Tehran, Iran.*

Let n be a square-free natural number. Let S be an infinite set of integer quadruples (a, b, c, d) such that the sets $\{ad - bc : (a, b, c, d) \in S\}$ and $\{ac - nbd : (a, b, c, d) \in S\}$ are bounded. Prove that the set $\{a^2 - nb^2 : (a, b, c, d) \in S\}$ is bounded.

Editor's Note. As pointed out by several solvers, the statement above must be corrected by adding the hypothesis that at least one of c, d be nonzero for every quadruple $(a, b, c, d) \in S$. We apologize for this omission.