2008. *Proposed by Michael W. Botsko, Saint Vincent College, Latrobe, PA.*

Is there a function $f : \mathbb{R} \to (0, \infty)$ such that the inequality

$$\frac{f(x)}{f(y)} \leq |x - y|$$

holds for all real numbers x, y such that x is irrational and y rational?

2009. *Proposed by Greg Oman, University of Colorado, Colorado Springs, CO.*

Consider any metric space (X, d). Let $\mathcal{F}_X$ be the collection of all functions $f : X \to \mathbb{R}$ (not necessarily continuous). The set $\mathcal{F}_X$ possesses natural operations of addition and multiplication, namely the sum $f + g$ and product fg of two elements f, g of $\mathcal{F}_X$ are characterized by the identities

$$(f + g)(x) = f(x) + g(x) \qquad \text{and} \qquad (fg)(x) = f(x)g(x), \qquad \text{for all } x \in X.$$

Endowed with these operations, $\mathcal{F}_X$ is a ring. Since the sum and product of continuous real functions are continuous, the set $\mathcal{C}_X$ consisting of all continuous functions in $\mathcal{F}_X$ is a subring of $\mathcal{F}_X$. Is there a metric space (X, d) such that $\mathcal{C}_X$ isomorphic to $\mathcal{F}_X$ as rings, but $\mathcal{C}_X$ is a proper subset of $\mathcal{F}_X$?

2010. *Proposed by Mehtaab Sawhney (student), University of Pennsylvania, Philadelphia, PA.*

Let k be a positive integer. Consider the experiment of choosing a permutation π of k objects uniformly at random (i. e., any two permutations σ, π are equally likely to be chosen). Let N be the number of cycles of π. Find the expected value $\mathbf{E}[N2^N]$ of the random variable $N2^N$, as a function of k.

Quickies

Answers to the Quickies are on page 385.

1065. *Proposed by Adrian Chu (student), The Chinese University of Hong Kong, Hong Kong.*

Does the equation

$$a^2 + b^7 + c^{13} + d^{14} = e^{15}$$

have a solution in positive integers a, b, c, d, e?

1066. *Proposed by Ovidiu Furdui, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.*

Is there a real matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ whose natural exponential

$$\exp(A) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{A}{1!} + \frac{A^2}{2!} + \cdots + \frac{A^n}{n!} + \ldots$$

satisfies the equation $\exp(A) = \begin{pmatrix} e^a & e^b \\ e^c & e^d \end{pmatrix}$?