Problem Section

Editor

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This section features problems for students at the undergraduate and (challenging) high school levels. Problems designated by "S" are especially well-suited for students. All problems and/or solutions should be submitted to Andy Liu, Mathematics Department, University of Alberta, Edmonton, Alberta T6G 2G1, Canada. Electronic submissions may also be sent to `aliu@math.ualberta.ca`. Please include your name, email address, school affiliation, and indicate if you are a student.

Proposals

To be considered for publication, solutions to the following problems should be received by November 10, 2006.

S107. *Correction.* Place the sixteen non-pawn pieces of a standard chess set on a 5×5 chessboard such that no piece attacks a piece of the opposite color. *The two Bishops of each color must stand on squares of opposite colors.*

S108. *Clarification.* A piece may *not* simply move to an adjacent vacant space. It *must* push someone else into a vacant space immediately beyond.

S110. *Proposed by J. L. Díaz-Barrero, Universitat Politècnica de Catalunya, Barcelona.* Let θ be a real number such that $0 < \theta < \pi/2$. Prove that

$$\sqrt{\sin^2\theta + \frac{1}{\sin^2\theta}} + \sqrt{\cos^2\theta + \frac{1}{\cos^2\theta}} \geq \sqrt{10},$$

and determine when equality holds.

S111. *Proposed by Linda Yu, Montreal.* A chess King can move from square to adjacent square, orthogonally or diagonally. He goes on a closed tour visiting all 64 squares of a standard chessboard, and his path does not cross itself. What is the maximum number of diagonal moves he may make?

S112. *Proposed by Mircea Ghita, Flushing, New York.* Inside a triangle, there are three circles each of which is tangent to two sides and the incircle of the triangle. Let their radii be r_1, r_2, and r_3. Prove that $\sqrt{r_1r_2} + \sqrt{r_2r_3} + \sqrt{r_3r_1} = r$, where r is the inradius of the triangle.

Problem 203. *Proposed by Greg Oman, Ohio State University.* Prove that for any odd prime number p, there exists a unique positive integer n such that $n(p + n)$ is the square of an integer.

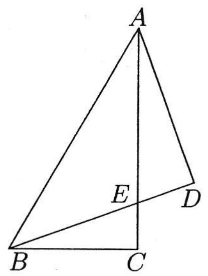
Problem 204. *Proposed by James Camacho Jr., Jersey City State College, and Jonathan Strzelec, student, Christopher Newport University.* Evaluate (a) $\int \sqrt{\tan\theta}\, d\theta$; (b) $\int_0^{\pi} \frac{\theta \sin\theta}{1+\cos^4\theta}\, d\theta$.

Solutions:

S104. A Trigonometric Identity. *Proposed by Stanley Rabinowitz.* Prove that

$$\tan 20° + 4 \sin 20° = \sqrt{3}.$$

Solution by Jean Huang, student, Harvard College. Let BAC be a triangle with $\angle ABC = 60°$ and $\angle CAB = 30°$. Let BAD be a triangle with $\angle ABD = 40°$ and $\angle DAB = 50°$. Moreover, let AC and BD intersect at E. Take $BC = 1$. Then $CA = \sqrt{3}$ and $AB = 2$. Hence $AD = AB \sin 40° = 4 \sin 20° \cos 20°$ and $AE = AD \sec 20° = 4 \sin 20°$. On the other hand, $CE = BC \tan 20° = \tan 20°$. It follows that $\tan 20° + 4 \sin 20° = CE + AE = AC = \sqrt{3}$.



Also solved by Brian Bradie, Cal Poly Pomona Problem Solving Group, James Camacho Jr., Alper Cay, Michael Faleski, Dmitri Fleischman, Allen Fuller, Carlos Gamez, Dipendra Bhattacharya with Stephen Glendon, Robert Gutierrez (graduate student), K. H. Ho with S. C. Ko with S. Y. Siu (high school students), Tylor London (student), David Lovit with Katherine Merow (students), Damien Mondragan (student), Lacey Moore (student), James Nielsen (student), Northwestern University Math Problem Solving Group, Ohio Wesleyan Problem Killers, Paolo Perfetti, David Swanson (student),