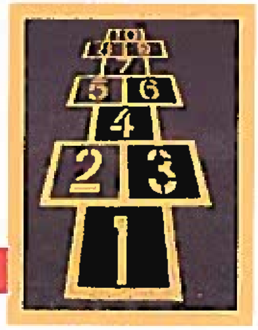
THE PLAYGROUND

Welcome to the Playground. Playground rules are posted on page 33, except for the most important one: *Have fun!*



THE SANDBOX

In this section, we highlight problems that anyone can play with, regardless of mathematical background. But just because these problems are easy to approach doesn't necessarily mean that they are easy to solve!

Problem 318. This problem comes to us from Greg Oman of the University of Colorado. In **Odd Job,** Greg asks you to find all positive integers n and k satisfying

$$1+3+5+\cdots+(2n-1)$$
$$=(2n+1)+(2n+3)+(2n+5)+\cdots+(2n+(2k-1)).$$

Problem 319. Our second sandbox problem is another game for Alice and Bob, from my colleague at Lafayette College, Tom Yuster. In **No Roots for You,** Alice and Bob alternate turns, each one assigning a non-zero real number to one of the coefficients of a polynomial, with Alice going first. Once a value has been assigned to a coefficient, it cannot be changed. The game ends when all of the coefficients have been assigned. Alice wins if the resulting polynomial has no real roots, and Bob wins otherwise.

1) Suppose Alice and Bob play this game with the polynomial ax^2+bx+c. Alice might first assign $a=1$, then Bob could assign $c=-2$, and finally Alice assigns $b=3$. Then Bob wins because x^2+3x-2 has real roots. Assuming both players use optimal strategies, who will win? Describe that winning strategy.

2) Repeat part 1 for the quartic polynomial $ax^4+bx^3+cx^2+dx+e$.

THE ZIP-LINE

This section offers problems with connections to articles that appear in the magazine. Not all Zip-Line problems require you to read the corresponding article, but doing so can never hurt, of course.

Problem 320. Joshua Bowman's article about billiards paths in polygons gives us **Bank Shot.** In both parts of this problem, assume the ball is represented by a point that bounces off each wall with the angle of incidence equal to the angle of reflection and with no loss of energy.

1) A pool table has dimensions $a\times b$ for positive integers a and b, with holes in each of the four corners. A ball is shot from one corner of the table at a 45° angle. How many times will it bounce off a wall before it lands in a pocket?

2) Now try this in three dimensions: A box with dimensions $a\times b\times c$ (for positive integers a, b, and c) has holes in each of its eight corners, and a ball is shot from one corner along the line $x=y=z$. How many times will it bounce off a wall before it lands in a pocket?

THE JUNGLE GYM

Any type of problem may appear in the Jungle Gym—climb on!

Problem 321. Draw $n\geq 2$ lines in the plane, no two parallel, no three (or more) through a common point of intersection.

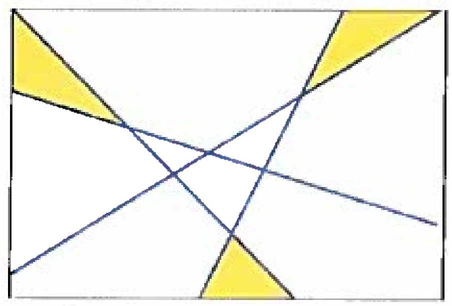


Figure 1.

This process will produce many regions in the plane (don't tell us how many—we already know). Call a region *trivial* if it is bounded by only two lines. (Such a region must be unbounded, of course.) The lines in figure 1 have three trivial regions. **Region Count** asks you to find the maximum and minimum numbers of trivial regions that n lines can create.

THE CAROUSEL
OLDIES BUT GOODIES

In this section, we present an old problem that we like so much, we thought it deserved another go-round. Try this, but be careful—old equipment can be dangerous. Answers appear at the end of the column.

Suppose $a_1, a_2, \ldots$ is a sequence of positive integers, where $a_n < a_{n+1}$ for all $n\geq 1$. Let u_n be the least common multiple of $(a_1,\ldots,a_n)$. Show that $\sum_{n=1}^{\infty}(1/u_n)$ converges.