939. *Proposed by Cezar Lupu (student), University of Bucharest, Bucharest, Romania.*

Let A be an $n \times n$ complex matrix. Let $\operatorname{adj}(A)$ be the adjugate of A, that is, the transpose of the matrix of the co-factors of A. Let $\operatorname{tr}(A)$ be the trace of A. Show that $\operatorname{tr}(\operatorname{adj}(A)^k) = 0$ for all k, $1 \le k \le n$, if and only if $\operatorname{adj}(A)^2$ is a zero matrix.

940. *Proposed by Greg Oman, Ohio University, Athens OH.*

Find all pairs (R, n) where R is a nonzero ring (not necessarily commutative or with identity) and $n > 1$ is an integer such that the following holds:

$$x_1x_2\cdots x_n \in \{x_1, x_2, \dots, x_n\}$$

for all $x_1, x_2, \dots, x_n \in R$.

SOLUTIONS

An externally trilinable 7-gon

911. *Proposed by Michael Scott McClendon, University of Central Oklahoma, Edmond OK.*

Given an n-gon P, a point x in the same plane as P is said to be *m-trilinable* if there exist 3 points x_1, x_2, and x_3 on P such that $d(x, x_1) = d(x, x_2) = d(x, x_3) = m$, where $d(a, b)$ is the Euclidean distance between points a and b. An n-gon is said to be *externally trilinable* if every point in the plane exterior to P is m-trilinable for some m. Is it possible for a 7-gon to be externally trilinable?

Solution by Darryl Nester, Bluffton University, Bluffton OH.

First note that the statement "x is m-trilinable" is equivalent to "a circle with radius m centered at x intersects P in at least three points."

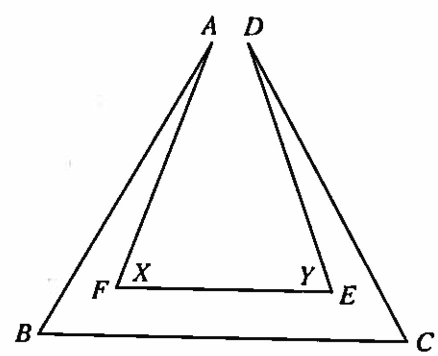

Figure 1. Non-convex hexagon

Let P be the non-convex hexagon $ABCDEF$ in Figure 1. Extend line segments $\overline{BA}$ and $\overline{CD}$ to meet at G, and $\overline{FA}$ and $\overline{ED}$ to meet at H. Then the exterior of P is equal to the union of the interior of ΔEFH and the exterior of ΔBCG. For a point Z in the interior of ΔEFH, we can clearly choose an appropriate radius so that a circle of that radius centered at Z will intersect P in at least three points. For example, if Z is above $\overline{AD}$, pick a point K in the interior of the quadrilateral $AFED$, then the circle centered at Z with radius ZK intersects P in at least three points. If Z is in the interior of the