PROBLEMS AND SOLUTIONS

EDITORS

Greg Oman
CMJ Problems
Department of Mathematics
University of Colorado, Colorado Springs
1425 Austin Bluffs Parkway
Colorado Springs, CO 80918
email: cmjproblems@maa.org

Charles N. Curtis
CMJ Solutions
Mathematics Department
Missouri Southern State University
3950 E Newman Road
Joplin, MO 64801
email: cmjsolutions@maa.org

This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively *both* by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

Proposed problems should be sent to **Greg Oman**, either by email (preferred) as a pdf, TEX, or Word attachment or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

Solutions to the problems in this issue should be sent to **Chip Curtis**, either by email as a pdf, TEX, or Word attachment (preferred) or by mail to the address provided above, no later than September 15, 2020. Sending both pdf and TEXfiles is ideal.

PROBLEMS

1171. *Proposed by George Apostolopoulos, Messolonghi, Greece.*

Let a, b, and c be the roots of the equation $x^3-2x^2-x+1=0$, with $a<b<c$. Find the value of the expression $(\frac{a}{b})^2+(\frac{b}{c})^2+(\frac{c}{a})^2$.

1172. *Proposed by Xiang-Qian Chang, MCPHS University, Boston, MA.*

Suppose that a function $y=y(x)$ satisfies the following first-order differential equation:

$$y'+x^6-x^4-2yx^3-3x^2+yx+y^2-1=0,$$

with initial value $y(0)=\sqrt{\frac{\pi}{2}}$, show that $y(x)\sim\frac{1+x^4}{x}$ as x tends to infinity.

1173. *Proposed by Greg Oman, University of Colorado Colorado Springs, Colorado Springs, CO.*

All rings in this problem are assumed commutative with identity. Now, let R and S be rings and suppose that R is a subring of S (we assume that the identity of R is the identity of S). An element $s\in S$ is *integral over* R if s is a root of a monic polynomial $f(x)\in R[x]$. If we set $\overline{R}:=\{s\in S : s \text{ is integral over } R\}$, then it is well-known that $\overline{R}$ is a subring of S containing R. The ring $\overline{R}$ is called the *integral closure of* R *in* S. In case $\overline{R}=S$, then we say that S *is integral over* R. For a ring R, let $R^\times$ denote the multiplicative group of units of R. Prove or disprove: for every infinite integral

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domain D_1, there exists an integral domain D_2 such that D_2 is integral over D_1 and $|D_2^\times| = |D_1|$ (that is, the set of units of D_2 has the same cardinality as that of D_1).

1174. *Proposed by George Stoica, New Brunswick, Canada.*

Let $a_1, \ldots, a_k$ and $b_1, \ldots, b_k$ be complex numbers which are not integers. Prove that the infinite product below converges if and only if $\sum_{i=1}^k a_i = \sum_{i=1}^k b_i$. What is the value of the product?

$$\prod_{n=1}^{\infty} \frac{(n-a_1)(n-a_2)\cdots(n-a_k)}{(n-b_1)(n-b_2)\cdots(n-b_k)}$$

1175. *Proposed by George Stoica, New Brunswick, Canada.*

Let F_1 and F_2 be distinct proper subfields of the field $\mathbb{R}$ of real numbers. Is there a field isomorphism $f\colon F_1 \to F_2$ preserving signs, that is, for all real x: $x \in F_1$ and $x > 0$ if and only if $f(x) \in F_2$, $f(x) > 0$?

