PROBLEMS AND SOLUTIONS

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This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively *both* by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

Proposed problems should be sent to **Greg Oman**, either by email (preferred) as a pdf, TEX, or Word attachment or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

Solutions to the problems in this issue should be sent to **Chip Curtis**, either by email as a pdf, TEX, or Word attachment (preferred) or by mail to the address provided above, no later than November 15, 2020. Sending both pdf and TEXfiles is ideal.

PROBLEMS

1176. *Proposed by Xiang-Qian Chang, MCPHS University, Boston, MA.*

Let $A_{n\times n}$ be an $n \times n$ positive semidefinite Hermitian matrix. Prove that the following inequality holds for any pair of integers $p \geq 1$ and $q \geq 0$:

$$\frac{\mathrm{Tr}(A^p) + \mathrm{Tr}(A^{p+1}) + \cdots + \mathrm{Tr}(A^{p+q})}{\mathrm{Tr}(A^{p+1}) + \mathrm{Tr}(A^{p+2}) + \cdots + \mathrm{Tr}(A^{p+q+1})} \leq \frac{r_A}{\mathrm{Tr}(A)},$$

where r_A is the rank of A and Tr is the trace function.

1177. *Proposed by Ovidiu Furdui and Alina Sîntămărian, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.*

Find all continuous functions $f\colon \mathbb{R} \to \mathbb{R}$ which satisfy the following equation for all $x \in \mathbb{R}$:

$$f(-x) = 1 + \int_0^x \sin t f(x-t)dt.$$

1178. *Proposed by Cezar Lupu, Texas Tech University, Lubbock TX, and Vlad Matei, University of Calirfornia Irvine, Irvine, CA.*

Consider a triangle ABC. Let $\mathcal{C}$ be the circumcircle of ABC, r the radius of the incircle, and R the radius of $\mathcal{C}$. Let $\mathrm{arc}(BC)$ be the arc of $\mathcal{C}$ opposite A, and define

[0]http://dx.doi.org/10.4169/college.math.j.51.3.xxx

$\mathrm{arc}(CA)$ and $\mathrm{arc}(AB)$ similarly. Let $\mathcal{C}_A$ be the circle tangent internally to the sides AB, AC, and the arc BC not containing A, and let R_A be its radius. Define $\mathcal{C}_B$, $\mathcal{C}_C$, R_B, and R_C similarly. Prove that the following inequality holds:

$$4r \leq R_A + R_B + R_C \leq 2R.$$

1179. *Proposed by Greg Oman, University of Colorado, Colorado Springs, Colorado Springs, CO.*

Let R be a ring, and let I be an ideal of R. Say that I is *small* provided $|I| < |R|$ (that is, I has smaller cardinality than R). Suppose now that R is an infinite commutative ring with identity which is not a field. Suppose further that R possesses a small maximal ideal M_0. Prove the following:

1. there exists a maximal ideal M_1 of R such that $M_1 \neq M_0$, and
2. M_0 is the *unique* small maximal ideal of R.

1180. *Proposed by Luke Harmon, University of Colorado, Colorado Springs, Colorado Springs, CO.*

In both parts, R denotes a commutative ring with identity. Prove or disprove the following:

1. there exists a ring R with infinitely many ideals with the property that every nonzero ideal of R is a subset of but finitely many ideals of R, and
2. there exists a ring R with infinitely many ideals with the property that every proper ideal contains (as a subset) but finitely many ideals of R.

