Problem Proposal #77

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Problem. Let R be a commutative ring (not assumed to have an identity). Recall that an element $x \in R$ is a *zero divisor* if there is some nonzero $y \in R$ such that $xy = 0$; x is nilpotent if $x^n = 0$ for some positive integer n (note that we do *not* require a zero divisor to be nonzero).

(a) Prove or disprove: there exists a finite commutative ring R for which

(1) every element of R is a zero divisor, and
(2) the only nilpotent element of R is 0.

(b) Does your answer change if "finite" is replaced with "infinite"?