MATH 2150 Autumn 2022 Lecture 1

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(4) "$x + y = 4$" is NOT a proposition. The issue here is that we are not given what x and y are. If x and y both happen to be 2, then this is a true proposition, but if $x = 6$ and $y = 3$, then the assertion is false. The issue here is that the assertion is not *unambiguously* true or false; its truth value depends on the value of x and y, and so this is NOT a proposition.

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(5) “This sentence is false” is not a proposition. This is a somewhat famous example of a so-called paradox. If the sentence is true, then the sentence is false, based on what the sentence is asserting. But if “This sentence is false” is false, then the sentence is true. We generally won’t be going into paradoxes in this course. If you are interested in reading more about these, you may want to look up the so-called “barber paradox”.

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REMARK: We may extrapolate the following more general principle from the previous two examples: If p is a true proposition, then $(\neg p)$ is false; if p is a false proposition, then $(\neg p)$ is true. We me express this more succinctly using the following true table.

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Truth table for the “not” operator:

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Let p be "$2 \times 3 = 6$" and q be "Canada is a continent". Note that p is true but q is false. Thus $(p \wedge q)$ is false precisely because it is NOT the case that p and q are BOTH true.

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Let p and q be propositions. Then the conditional proposition **if** p**, then** q, denoted $(p \to q)$ and read "if p, then q", is ONLY FALSE IF p is true and q is false. I CANNOT OVERSTATE HOW IMPORTANT IT IS FOR YOU TO COMMIT THIS TO MEMORY **NOW!!!**. The motivation here is that "if p then q" is saying that the truth of p, in some sense, *forces* the truth of q. If p happens to be false, then no forcing can happen, and in this case, we simply declare "if p, then q" to be true by fiat.

Example

Let p be "$2 - 5 = -3$" and q be "$0 \cdot 10 = 5$". Then note that p is true but q is false, and hence $(p \to q)$ is false.

Here is the truth table:

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p	q	$(p \rightarrow q)$
T	T	T
T	F	F
F	T	T
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Let p be "a cat is an insect" and q be "$2 + 3 = 5$".

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Example

Let p be "$11 = 4$" and q be "$2 + 3 = 89$". Then both p and q are false, and hence p and q have the same truth value, so $(p \leftrightarrow q)$ is true.

Here is the truth table:

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We have now seen the five fundamental logical operators: $\neg$ (not), $\wedge$ (and), $\vee$ (inclusive or), $\rightarrow$ (if, then), and $\leftrightarrow$ (if and only if).

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We have now seen the five fundamental logical operators: $\neg$ (not), $\wedge$ (and), $\vee$ (inclusive or), $\rightarrow$ (if, then), and $\leftrightarrow$ (if and only if). Some of you may have seen other logical operators in a cs course. These operators can be expressed using only the logical operators above (end of lecture).