A group in the PROMYS Math Circle Girls (PMCG) initiative at Boston University working on a problem. The PMCG initiative was awarded a Tensor Women grant in 2019.

A Life in Search of Problems

Alicia Prieto-Langarica

Does "problem solving" elicit visuals of the not-so-great parts of mathematics: 'cut-throat' competition, individualism, elitism, and isolation? Or are you an enthusiast for whom problem solving (and even competitive problem solving) brings a sense of community, belonging, and laughter? Featured in this issue, Julia Shanen's poem, "My First Time at the AMC (Not the One That Sells Popcorn)," beautifully shows how problem solving can lend itself to both competition and community. For some of us, problem solving was the inspiration to pursue higher education in mathematics.

Growing up, I thought mathematics was boring and mechanistic. All someone had to do to get good grades was memorize a multiplication table or apply the same algorithmic procedure seen in class to a VERY similar exercise. I did not see any appeal. However, the summer after sixth grade, while I was in a camp aimed at "remediating" those of us not yet ready for the next grade, a teacher challenged us with a problem. It was dramatically different from anything I had seen, and it completely changed the trajectory of my academic career.

The problem went something like this: An astronaut is taking a space walk. When she comes back, she wants to fold the long cord she used for her walk. She knows the length of the cord in feet is an integer number smaller than 100. She tries folding it 2×2 feet, but she notices she has an extra foot. She tries folding it 3×3 feet and she has 2 extra feet and so on until 6×6 feet give her 5 extra feet. What is the length of the cord?

If you know basic algebra (or even some congruences), this problem is simple. However, for my 13-year-old self, this was a challenging, but fun puzzle. Questions like this can be used to inspire students to want to learn more. Without a doubt, I became a mathematician because of this experience. I am not alone in having been tremendously inspired by a good problem. Dan Ullman, problems editor for the *American Mathematical Monthly*, shared with us: "(Growing up), I was very lucky that Murray Klamkin, the legendary

problemist, was a close friend of my father and was around my house regularly when I was a small child. He was always giving me math puzzles, and I attribute my enjoyment of problems to his influence."

A similar story comes from Les Reid, problems editor for *Mathematics Magazine,* who was given challenging questions by his high school teacher and math team coach, Andre Samson. For practice, Andre would give the team problems that were even more challenging than the ones they encountered in competition. Encouraged by his teacher, Les submitted a solution to a problem from the *Monthly* in 1973 and received positive feedback from the editor. This led him to compete in the second U.S.A. Mathematical Olympiad in 1973 and in the Putnam Competition in college.

Dan Ullman and Les Reid are just two of the four MAA journal problems editors we were eager to hear from on their passion for brain teasers. We also spoke with Greg Oman, the problems editor of the *College Mathematics Journal*, and Glen Whitney, outgoing editor of The Playground in *Math Horizons*, about their perspectives on problem solving. We want to honor their important work, which inspires so many others to play with mathematical ideas.

So, what makes a "good" problem?

> The best problems make you smile or even laugh. A good problem is like a good joke. The problem statement is the set-up, and the solution is the punch line. —Dan Ullman

What makes a good problem is a matter of personal taste. A problem is "good" if it is interesting, and challenging to you. So, the answer depends on whom you ask. For Glen Whitney, good problems are those he can "touch" or "see." Glen has a soft spot for geometry problems or for problems with interesting graphs or visual representations. He also appreciates problems that yield to a slightly different way of looking at them than the obvious way.

Greg Oman, an algebraist and logician by trade, is drawn to problems that are algebraic in nature. Greg believes that a good question will appeal to a wide variety of readers. It will require more than simply rote methods and techniques to solve. According to Greg, the best problems connect different areas of mathematics, are easy to understand, and motivate the would-be solver to find a solution.

While they each have their own personal tastes, all four of our MAA problems editors agree that the best problems are those for which—even if a very technical solution exists—there is also at least one solution that requires little theoretical background but which relies more on creativity and out-of-the-box thinking. Those are the solutions that leave the solver wanting more and that open up one's curiosity to learn the theoretical background needed to understand the more complicated solution.

Greg Oman is an associate professor of mathematics at The University of Colorado, Colorado Springs. When not doing math, he enjoys hiking, traveling, and mentoring research students.

Les Reid was born in South Dakota and moved to the northern Virginia suburbs of Washington, DC when he was twelve. He received his bachelor's and master's degrees from the University of Chicago, worked as a mathematical analyst in industry for a year, and then went to Duke University for graduate school, where he earned his PhD. Les spent four years as a postdoc at the University of Utah. He is now a professor at Missouri State University. His five-year old twin boys keep him busy (and delight him).

Daniel Ullman received his BA from Harvard in 1979 and his PhD from Berkeley in 1985. He has been on the faculty at George Washington University ever since. He was the American Mathematical Society Congressional Fellow in 2006–2007. He has been involved with the Problems Section of the *American Mathematical Monthly* since the early 1990s, serving as lead editor of the section from 1997 to 2001 and then again from 2017 to the present. He is the director of the Putnam Competition. His hobbies include playing the piano, playing soccer and racquetball, and playing Scrabble and other board games.

Glen Whitney received a PhD in logic from UCLA, but his initial career as a pure math academic veered into applied math and finance when he joined a hedge fund. A decade later, he shifted direction again into public math outreach, founding the National Museum of Mathematics. As one component of public engagement, Glen has served as the editor of the "Varsity Math" column in the *Wall Street Journal*, "The Playground" in *Math Horizons*, and now *The Prisoner's Dilemma* (prisonersdilemma.org) for the Prison Math Project.

How does one go about writing one of these "good" problems?

Greg Oman subscribes to Terence Tao's approach: ask yourself dumb questions and then attempt to answer them. Often this involves translating results from one mathematical set-

Student working on the AMC 8 in Salisbury, England.

ting to see what's true in another. Or it might involve applying mathematics to situations we encounter out in the world. Oman recalls a beautiful example from the 2004 Putnam exam that he calls the "Free Throw Intermediate Value Problem (FTIVP)."

> *Bsketball star Shanille O'Keal's team statistician keeps track of the number, S(N), of successful free throws she has made in her first N attempts of the season. Early in the season, S(N) was less than 80% of N, but by the end of the season, S(N) was more than 80% of N. Was there necessarily a moment in between when S(N) was exactly 80% of N?*

For Glen Whitney, the process of writing good problems is a bit more "heuristic." While doing routine things, he tends to think about the processes involved in this routine. He gives us a fantastic example in the form of a problem he wrote during his time as *Playground* editor, inspired by actual events.

> *Each day, Professor Kibacha takes a half a pill as follows: she dumps one item out of the bottle (uniformly randomly selected). If it's a half pill, she swallows it; otherwise, she breaks it, takes half, and puts the other half back. One day, a whole pill shakes out but drops down the drain. After counting the whole and half pills remaining, the professor thinks, "Hmm, losing that pill did not change the expected number of half pills in the bottle when I end up breaking the last whole pill." What is the difference between the number of whole pills and half pills left in the bottle?*
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> —Glen Whitney (2018) The Playground, *Math Horizons*, 26:3, 30–33, doi.org/10.1080/10724117.2018.1558619

Glen says a problem can be good just by being a cute question and having a nice setup. The answer doesn't always have to come out slickly or nicely. But to reach the highest heights of a great problem, they should have a slick solution. Glen thinks of lots of random questions to try as he goes about his daily life, and then, he says, sometimes he gets lucky—something he comes up with actually works out to have a nice answer or there's a creative way to solve it.

What do the MAA problems editors want people to take away from answering these questions?

The four MAA problems editors agree that the main reason to play with puzzles is because it's fun. But it goes beyond that. As Les Reid points out, solving these problems can help students better understand a concept or a technique. Both Greg Oman and Glen Whitney compare problem solving to mathematical research. Greg says, "We researchers typically spend most of our time stuck, and it often requires tenacity and perseverance to make progress. Problem solving is a good model for this."

Glen points out that working on a challenging puzzle helps students feel comfortable with making mistakes. They can try every path and every technique. No one expects or requires students to solve these problems, so the students are at liberty to try different things and to experiment. Unlike homework problems, for which students typically know which techniques are being emphasized, students initially have no idea regarding how a problem could be approached. Perhaps their first five or six different ways of trying to solve it aren't going to work out. In that sense, it's a little closer to what mathematics research is like.

Dan Ullman hopes that the problems sections of all MAA journals invite people—especially students—to enjoy themselves. It feels good to engage in a mental activity that is really challenging. It turns out that working problems builds mathematical skill and creativity, which means that this joyous activity is actually good for you. The world needs more people who are willing to tackle difficult questions and who can contribute to their solutions creatively.

As a professor, I have worked on problems in the MAA journals with my students. When working on such difficult questions, I am no longer the expert PhD with all the knowledge. My students and I are equal, and we can get stuck and try different things together. This is so empowering to students. It's also an important reminder to me that mathematical creativity can be found anywhere and at any stage. We are all capable of playing with intriguing questions.

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