MATH 2150 Autumn 2022 Lecture 7

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Example

Let $P(x, y)$ be "$x + 1 = y$".

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Let $P(x, y)$ be "$x + 1 = y$". Then $P(x, y)$ is a two-place predicate.

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The following rules govern the construction of formulas in predicate logic:

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The following rules govern the construction of formulas in predicate logic:

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[1]Thus x NEVER occurs free in $\forall x\alpha$ or $\exists x\alpha$.

n-Place Predicates

We now give a recursive definition of "freeness" analogous to our definition in the one-place setting. Again, the process is similar here.

Definition (Recursive definition of "freeness" for general formulas)

Let x and y be variables (with repetitions allowed) and α and β be formulas.

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4. x occurs free in $\exists y\alpha$ exactly when x occurs free in α and $x \neq y$, and

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Solution x occurs free in $Q(x, y, z)$ by 1.

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Example

Justify why y occurs free in $(P(x, z) \to \exists x(Q(x) \vee Q(y)))$.

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Domains and Translating I

In this subsection, we will discuss domains (again, but for *n*-place predicates more generally) and introduce translating from formulas to English and vice-versa.

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Consider the 3-place predicate $P(x, y, z) =$ "$x + y = z$". Then reasonable domains for x, y, and z would be the set of real numbers (or the set of integers, or the set of complex numbers, or the set of positive real numbers, etc.).

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Domains and Translating I

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Solution Remember the algorithm: "For all (what kind of object x is) x, $P(x)$.", which becomes "For all real numbers x, $x > 1$."

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Domains and Translating I

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Let $P(x)$ be "x is tall" and let $Q(y)$ be "y is old". Further, let the domain for both x and y be the collection of all people. Translate $\forall x \exists y (P(x) \land Q(y))$ into English.

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Let $P(x)$ be "x is tall" and let $Q(y)$ be "y is old". Further, let the domain for both x and y be the collection of all people. Translate $\forall x \exists y (P(x) \wedge Q(y))$ into English.

Solution The general algorithm here is "For every (what kind of object x is) x, there exists (whatever kind of object y is) y such that $P(x)$ and $Q(y)$.", which becomes "For every person x, there exists a person y such that x is tall and y is old." □

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Solution The algorithm, more generally, is "For every (what kind of object x is) x, there exists (what kind of object y is) y and there exists (what kind of object z is) z such that $P(x, y, z)$." which becomes "For every integer x, there exists an integer y and there exists a real number z such that $x + y < z$." $\square$

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IMPORTANT!!! If you are translating a formula which contains two consecutive existential quantifiers, the way to do this is to separate the two "there exists" with an "and" and write the "such that" *only* after the second existentially quantified object; do NOT write it twice (see above example and you should understand what I mean by this)

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Let $Q(a, b, c)$ be "a hires b at c"; let the domain for a be the set of all executives, the domain for b to be the set of all secretaries, and the domain for c be the set of all companies.

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Domains and Translating I

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Let $R(x, y, z)$ be "$\frac{x}{y} = z$" and let the domain for x and z be the set of all real numbers and the domain for y be the set of all nonzero real numbers.

Domains and Translating I

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Solution "For every real number x and for every nonzero real number y, there exists a real number z such that $\frac{x}{y} = z$." □

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Domains and Translating I

Example

Let $R(x, y, z)$ be "$\frac{x}{y} = z$" and let the domain for x and z be the set of all real numbers and the domain for y be the set of all nonzero real numbers. Translate $\forall x \forall y \exists z R(x, y, z)$.

Solution "For every real number x and for every nonzero real number y, there exists a real number z such that $\frac{x}{y} = z$." □

IMPORTANT!!! When translating a formula which contains two consecutive universal quantifiers, insert an "and" in between as with existential quantifiers.

Example

Let $P(x, y)$ be "x and y are even" and $Q(x, y)$ be "x and y are odd". Translate $\forall x \forall y (P(x, y) \rightarrow (\neg Q(x, y)))$, where the domain for x and the domain for y is the set of all integers.

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Finally, as with translations in the one-place predicate realm, any translation of an English sentence to a formula will NEVER have ANY occurrence of a free variable.