Math 2150 Lecture 18

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Intro to Sets

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Again, if the answer were "yes", then by definition, every member of the set on the left would be a member of the set on the right. Observe that $\{1\}$ is a member of the set on the left.

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Again, if the answer were "yes", then by definition, every member of the set on the left would be a member of the set on the right. Observe that $\{1\}$ is a member of the set on the left. So if the answer were yes, then we would also have that $\{1\} \in \{1\}$.

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The reason for including this example is to illustrate a point: DON'T TRUST YOUR FEELINGS ON PROBLEMS LIKE THIS; TRUST LOGIC AND DEFINITIONS! I cannot overemphasize this point. The first thing I want to point out is that there is only one number 1.

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Example

Let A be any set. What is $\varnothing \backslash A$?

Solution You may feel a bit fuzzy about this one, since I didn't explicitly define the set A for you. As I said in the previous lecture notes, this is where you need to use definitions to get your bearings. What are the members of $\varnothing \backslash A$? By definition, they are the objects that are in $\varnothing$ which are not in A. Are there any such objects? If there is one, then that object is in $\varnothing$, which is impossible since $\varnothing$ contains no elements at all! Thus $\varnothing \backslash A$ contains no members, i.e., $\varnothing \backslash A = \varnothing$.

Intro to Sets

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