Math 2150 Lecture 20

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Finishing Sets

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Case 2: $x \in B \backslash A$. Then $x \in B$ and $x \notin A$, and therefore $x \in B$, as was to be shown. $\square$

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(i) Assume that $A = B$. We must prove that $\mathcal{P}(A) = \mathcal{P}(B)$. What we must prove is that for all objects x: $x \in \mathcal{P}(A)$ iff $x \in \mathcal{P}(B)$. So let x be an arbitrary object. Again, remember that the way you prove iff statements is to assume one of the "pieces" and deduce the other, then assume the other piece and deduce the first. □

Finishing Sets

Proof.

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5. $\mathbb{C} = \{a + bi \colon a, b \in \mathbb{R}\}$, the set of **complex numbers**.