Problem #134

Problem. Let R be a commutative ring with identity $1 \neq 0$. For the purposes of this problem, define the *non-unit covering number* of R to be the smallest cardinal number κ such that the set of non-units of R can be expressed as the union of κ-many ideals. Prove or disprove the following:

(1) If either R has but finitely many maximal ideals or R has a finite non-unit covering number, then the non-unit covering number of R is the same as the cardinality of the collection of maximal ideals of R.

(2) If either R has countably many maximal ideals or R has covering number at most $\aleph_0$, then the non-unit covering number of R is the same as the cardinality of the collection of maximal ideals of R.