

Problem. Let S be a nonempty set and suppose that $\mathcal{F}$ is a nonempty collection of functions $f: S \rightarrow S$. For a subset $X \subseteq S$, say that X is *closed under $\mathcal{F}$* if for every $x \in X$ and $f \in \mathcal{F}$, $f(x) \in X$. Say that $(S, \mathcal{F})$ is *simple* if the only subsets of S that are closed under $\mathcal{F}$ are $\emptyset$ and S . For any non-empty set S , let $\nu(S)$ be the smallest cardinality of a nonempty collection $\mathcal{F}$ of functions on S such that $(S, \mathcal{F})$ is simple. Determine $\nu(S)$ as a function of $|S|$.