Problem Proposal #1286 (Fiala and Oman)

Problem. Let D be an integral domain, that is, a commutative ring with identity $1_D \neq 0$ without zero divisors. A *subring* of D is a nonempty subset of D closed under addition, multiplication, and negatives (note that in this problem, we do not require a subring to contain 1_D). Prove that every subring of D has an identity (a priori, different subrings may contain different identities) if and only if $D := F$ is a field of prime characteristic p and F is algebraic over its prime subfield $\mathbb{F}_p := \{m \cdot 1_D \colon m \in \mathbb{Z}\}$.