MATH 2150 Autumn 2022 Lecture 2

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Formulas and Syntax

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Formulas and Syntax

Definition (recursive definition of formulas)

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2. If α and β are formulas, then so are $(\neg\alpha)$, $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$, $(\alpha \to \beta)$, and $(\alpha \leftrightarrow \beta)$.

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In the next examples, we justify why certain expressions are formulas. You will also have a bit of this to do in the homework, just fyi.

Formulas and Syntax

Example

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Example

Let p be a propositional variable. Then $(\neg(\neg p))$ is a formula.

Since p is a propositional variable, p is a formula. Now applying 2., we see that $(\neg p)$ is a formula. By 2. again (with α being $(\neg p)$), we see that $(\neg(\neg p))$ is a formula.

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To see why this is true, note first that p and q are formulas by 1. Now by 2., $(p \vee q)$ is a formula. By 1., r is a formula. Finally, by 2. (with α being $(p \vee q)$ and β being r), it follows that $((p \vee q) \to r)$ is a formula.

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Remark

The general idea is as follows: begin with the variables, and then "build up" the formula from the inside out, making sure to justify each step along the way by appealing to definitions. We will see more examples shortly.

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(IMPORTANT) The text indicates certain abbreviations which allows one to not include so many parentheses. I do NOT want you to do this. I want you to follow the algorithm precisely. The reason is simple: many of you will be doing some coding either in computer science, math, or engineering, and if you can't get the syntax right, you're likely to get garbage as output.

Semantics and more on truth tables

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First, consider propositional variables p and q. *Without context*, it doesn't make sense to say that, say, $(p \vee q)$ is *true*; similarly, it makes no sense to say that $(p \vee q)$ is *false*. For example, suppose that p is "$2 + 2 = 5$" and q is "Thomas Jefferson was the first president of the United States".

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Note the word "subformula" above. For this course, you may take "subformula" to mean simply a formula you come to as you justify (using the recursive definition) that a given formula really is a formula.

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Note the word "subformula" above. For this course, you may take "subformula" to mean simply a formula you come to as you justify (using the recursive definition) that a given formula really is a formula. Before diving into truth tables, let's look at another example.

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Let p and q be propositional variables. Then $((\neg p) \vee q)$ is a formula.

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To see why, observe that p and q are variables, hence formulas by 1. of the definition of “formula”. By 2. of the definition, $(\neg p)$ is a formula.

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Let p and q be propositional variables. Then $((\neg p) \vee q)$ is a formula.

To see why, observe that p and q are variables, hence formulas by 1. of the definition of "formula". By 2. of the definition, $(\neg p)$ is a formula. Now we have that both $(\neg p)$ and q are formulas, so by 2. again, $((\neg p) \vee q)$ is a formula.

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Example

p	q	$(\neg p)$	$((\neg p) \vee q)$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

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To see why, observe that p and q are variables, hence formulas by 1. of the definition of "formula". By 2. of the definition, $(\neg p)$ is a formula. Now we have that both $(\neg p)$ and q are formulas, so by 2. again, $((\neg p) \vee q)$ is a formula. We now use our work above to construct a truth table for $((\neg p) \vee q)$:

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Let p, q, and r be propositional variables. Then $((p \vee q) \wedge r)$ is a formula.

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Let p, q, and r be propositional variables. Then $((p \vee q) \wedge r)$ is a formula.

p, q, and r are formulas by 1. By 2., $(p \vee q)$ is a formula. By 2., $((p \vee q) \wedge r)$ is a formula.

Example (truth table)

p	q	r	$(p \vee q)$	$((p \vee q) \wedge r)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	T	F	F
T	T	F	T	F
T	F	F	T	F
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1. The number of rows of truth values in your table is equal to the number 2 raised the power which is the number of variables. Note that our last example had 3 variables, and $2^3 = 8$, which is the number of rows in the table.
2. It does NOT MATTER in what order your fill in the truth values under the variables; all you need to do is make sure that you list all possibilities (and don't repeat...)

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2. It does NOT MATTER in what order your fill in the truth values under the variables; all you need to do is make sure that you list all possibilities (and don't repeat...)

Let's conclude with a final example, which will allow us to transition into next week's material.

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$((p \vee (\neg p)) \vee q)$ is a formula.

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Both p and q are formulas by 1. of the definition.

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Both p and q are formulas by 1. of the definition. By 2., $(\neg p)$ is a formula.

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Both p and q are formulas by 1. of the definition. By 2., $(\neg p)$ is a formula. By 2. again, $(p \vee (\neg p))$ is a formula.

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$((p \vee (\neg p)) \vee q)$ is a formula.

Both p and q are formulas by 1. of the definition. By 2., $(\neg p)$ is a formula. By 2. again, $(p \vee (\neg p))$ is a formula. Finally, by 2., we see that $((p \vee (\neg p)) \vee q)$ is a formula.

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Example (truth table)

p	q	$(\neg p)$	$(p \vee (\neg p))$	$((p \vee (\neg p)) \vee q)$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
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Example

$((p \vee (\neg p)) \vee q)$ is a formula.

Both p and q are formulas by 1. of the definition. By 2., $(\neg p)$ is a formula. By 2. again, $(p \vee (\neg p))$ is a formula. Finally, by 2., we see that $((p \vee (\neg p)) \vee q)$ is a formula. And here's the truth table:

Example (truth table)

p	q	$(\neg p)$	$(p \vee (\neg p))$	$((p \vee (\neg p)) \vee q)$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	T	T

Semantics and more on truth tables

Remark

Note that the above formula is true *regardless of the truth values of the propositional variables*.

Semantics and more on truth tables

Remark

Note that the above formula is true *regardless of the truth values of the propositional variables*. This is an example of a formula called a **tautology**, which we will discuss next lecture.