MATH 2150 Autumn 2022 Lecture 4

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Colorado Springs

Translations and Negations in Propositional Logic

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Translations and Negations in Propositional Logic

We now revisit implications and biconditionals ("if then" and "if and only if"). Recall that $(p \to q)$ is only false when p is true and q is false; $(p \leftrightarrow q)$ is only true when p and q have the same truth values.

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Remark

You don't need to show me work on homework problems similar to this; I showed the work above because I'm teaching and want to be as helpful as possible.