MATH 2150 Lecture 11

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Intro to Proofs

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Definition (Direct Proof)

To prove a statement of the form $\forall x_1 \forall x_2 \cdots \forall x_n (P(x_1, x_2, \ldots, x_n) \rightarrow Q(x_1, x_2, \ldots, x_n))$ directly, do the following:

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Note that EVERY LETTER THAT WAS INTRODUCED IN THE PROOF WAS DESCRIBED – FOR EXAMPLE, I DIDN'T JUST WRITE "x" and "y" above; I stated that they were integers. In general, every letter introduced in a proof should be described as above. Also, note the use of COMPLETE SENTENCES. Next, observe that to be even, it is not enough to merely be $2\cdot$ some number; the "some number" must be an INTEGER. This is the reason for justifying that $n + m$ above is an integer: it must be in order for $x + y$ to be even.

Intro to Proofs

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Let x and y be arbitrary integers. Assume that x is even and y is odd. We must show that $x + y$ is odd. Because x is even and y is odd, $x = 2n$ for some integer n and $y = 2m + 1$ for some integer m.

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Remark

Again, observe that we want to justify that $n + m$ is an **integer** because the definition of "odd" requires us to be two times an integer plus one.

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