MATH 2150 Lecture 9

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Logical Equivalence in Predicate Logic

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Let α and β be formulas in predicate logic. Then α and β are **logically equivalent**, denoted by $\alpha \equiv \beta$, exactly when regardless of the interpretation of the predicates which appear in α or β and regardless of the domains of the variables appearing in α or β, α and β have the same truth value.

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Solution To show that two formulas are NOT logically equivalent, you want to find an interpretation (definition) of $P(x)$ as well as a domain for x which makes one formula true and the other false.

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Solution To show that two formulas are NOT logically equivalent, you want to find an interpretation (definition) of $P(x)$ as well as a domain for x which makes one formula true and the other false. Let $P(x)$ be "x is over 6 feet tall" and let the domain for x be the collection of all people.

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Let α and β be formulas and let x be a variable.

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Let $C(x, y)$ be "x and y have chatted online" and $I(x)$ be "x has an internet connection". Translate "Everyone with an internet connection has chatted with Bob" into a formula, negate as above, and then translate the negation to English.

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Solution The first part is "For every person x, IF x has an internet connection, THEN x has chatted with Bob", which becomes $\forall x(I(x) \rightarrow C(x, Bob))$.

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Solution The first part is "For every person x, IF x has an internet connection, THEN x has chatted with Bob", which becomes $\forall x(I(x) \rightarrow C(x, Bob))$. The negation is $\exists x(\neg(I(x) \rightarrow C(x, Bob))) \equiv \exists x(I(x) \wedge (\neg C(x, Bob)))$.

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Solution The first part is "For every person x, IF x has an internet connection, THEN x has chatted with Bob", which becomes $\forall x(I(x) \rightarrow C(x, Bob))$. The negation is $\exists x(\neg(I(x) \rightarrow C(x, Bob))) \equiv \exists x(I(x) \wedge (\neg C(x, Bob)))$. In English, this becomes "There exists a person x such that x has an internet connection and x has not chatted with Bob over the internet" or more simply, "There is a person with an internet connection that hasn't chatted with Bob over the internet". $\square$

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Solution The first part is "For every person x, IF x has an internet connection, THEN x has chatted with Bob", which becomes $\forall x(I(x) \to C(x, Bob))$. The negation is $\exists x(\neg(I(x) \to C(x, Bob))) \equiv \exists x(I(x) \land (\neg C(x, Bob)))$. In English, this becomes "There exists a person x such that x has an internet connection and x has not chatted with Bob over the internet" or more simply, "There is a person with an internet connection that hasn't chatted with Bob over the internet". $\square$

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