Math 2150 Assignment 1 Solutions

(1)[10 pts] Circle all of the following which are propositions. No justification is required.

1. Basketball is a professional sport in the United States.
2. Basketball is not a professional sport in the United States.
3. How many eggs are there?
4. There exists a blue unicorn.
5. $x = 2$.
6. Pikes Peak is a mountain in Colorado.
7. Why does the sun rise in the morning?
8. Every animal is the color green.
9. $14 = 7$.
10. President Biden has had his wisdom teeth out.

Solution. Both (1) and (2) are propositions, (3) is not, (4) is a proposition, (5) is not, (6) is a proposition, (7) is not, and (8), (9), and (10) are propositions. □

(2)[10 pts] Let p be "There is a concert tonight" and q be "There will be more arrests than usual tonight." Translate each of the following into English sentences. This problem should NOT require much thought. Just use the definitions in the notes and don't try to be fancy. (note: complete sentences end with punctuation (a period in this case))

1. $(\neg p)$
2. $(p \wedge q)$
3. $(p \vee q)$
4. $(p \to q)$
5. $(p \leftrightarrow q)$

Solution. Here are the translations (this is an easy problem, of course):

(1) There isn't a concert tonight.

(2) There is a concert tonight and there will be more arrests than usual tonight.

(3) There is a concert tonight or there will be more arrests than usual tonight.

(4) If there is a concert tonight, then there will be more arrests than usual tonight.

(5) There is a concert tonight if and only if there will be more arrests than usual tonight. □

(3)[10 pts] Let p be "There is a heat wave" and q be "There is a drought." Convert each of the following English sentences into formulas.

1. There is a heat wave and there is a drought.
2. If there is a heat wave, then there is a drought.
3. There isn't a drought.
4. There is a heat wave or there is a drought.
5. There is drought if and only if there is a heat wave.

Solution. Here are the translations:

(1) $(p \wedge q)$

(2) $(p \to q)$

(3) $(\neg q)$

(4) $(p \vee q)$

(5) $(q \leftrightarrow p)$ □

(4)[10 pts] For each of the following propositions, indicate if it is true or false. No justification is required. Let p be "$2 + 2 = 5$" and q be $2 \times 3 = 6$".

1. $(\neg p)$
2. $(p \wedge q)$
3. $(p \vee q)$
4. $(p \to q)$
5. $(p \leftrightarrow q)$

Solution. (1) is true, (2) is false, (3) is true, (4) is true, and (5) is false. □

(5)(a)[10 pts] Justify, as I did in the notes (using the inductive definition), that $(((\neg p) \vee r) \to q)$ is a formula.

Solution. p, q, and r are formulas by (1) of the definition. By (2) of the definition, $(\neg p)$ is a formula (since we already justified that p is a formula). By (2) again, $((\neg p) \vee r)$ is a formula (since we have justified that $(\neg p)$ and r are formulas). Finally by (2), we see that $(((\neg p) \vee r) \to q)$ is a formula (since we have already justified that $((\neg p) \vee r)$ and q are formulas). □

(5)(b)[10 pts] Give a truth table for $(((\neg p) \vee r) \to q)$ (just the table is fine; you don't need to justify anything).

Solution.

p	q	r	$(\neg p)$	$((\neg p) \vee r)$	$(((\neg p) \vee r) \to q)$
T	T	T	F	T	T
T	F	T	F	T	F
F	T	T	T	T	T
F	F	T	T	T	F
T	T	F	F	F	T
T	F	F	F	F	T
F	T	F	T	T	T
F	F	F	T	T	F

□

(6)(a)[10 pts] Justify that $((p \wedge q) \wedge (\neg r))$ is a formula.

Solution. p, q, and r are formulas by (1) of the definition. Now $(p \wedge q)$ is a formula by (2) of the definition (since we have justified that p and q are formulas). $(\neg r)$ is a formula by (2) of the definition (we have justified that r is a formula already). Finally, $((p \wedge q) \wedge (\neg r))$ is a formula by (2) (we have justified that $(p \wedge q)$ and $(\neg r)$ are formulas). □

(6)(b)[10 pts] Give a truth table for $((p \wedge q) \wedge (\neg r))$.

Solution.

p	q	r	$(\neg r)$	$(p \wedge q)$	$((p \wedge q) \wedge (\neg r))$
T	T	T	F	T	F
T	F	T	F	F	F
F	T	T	F	F	F
F	F	T	F	F	F
T	T	F	T	T	T
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	T	F	F

□