Math 2150 Assignment 2 Solutions

(1)[5 pts] Consider the formula $(p \to q)$, where p and q are propositional variables. Find specific propositions for p and q which make $(p \to q)$ true but the *inverse* $((\neg p) \to (\neg q))$ false. Please use simple mathematical assertions for p and q as I did in the notes (I did a problem just like this in the notes for the *converse*). Other than giving me the formulas, you do not need to justify your work.

Solution. Let p be "$0 = 1$" and q be "$1 = 1$". Then since p is false, $(p \to q)$ is true. But then $(\neg p)$ is true and $(\neg q)$ is false, which makes $((\neg p) \to (\neg q))$ false. $\square$

(2)[10 pts] Consider again the formula $(p \to q)$. Recall that the *contrapositive* of this conditional is the formula $((\neg q) \to (\neg p))$. Is $(p \to q)$ logically equivalent to $((\neg q) \to (\neg p))$? Please use a truth table and follow the algorithm (and examples) I gave in the notes.

Solution.

p	q	$(\neg p)$	$(\neg q)$	$(p \to q)$	$((\neg q) \to (\neg p))$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Since the final two columns are equal, the two formulas are logically equivalent. $\square$

(3)[10 pts] Use a truth table to determine if the formula $(p \to (p \vee q))$ is a *tautology.*

Solution.

p	q	$(p \vee q)$	$(p \to (p \vee q))$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Since the final column consists of all Ts, the formula is a tautology. $\square$

(4)[10 pts] Use a truth table to determine if $((p \vee q) \rightarrow p)$ is a *contradiction.*

Solution.

p	q	$(p \vee q)$	$((p \vee q) \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

Since the final column doesn't consist of all Fs, the formula is not a contradiction. $\square$

(5)[10 pts] Use a truth table to show that the formulas $(\neg(p \vee q))$ and $((\neg p) \wedge (\neg q))$ are logically equivalent (this is the second Demorgan Law).

Solution.

p	q	$(\neg p)$	$(\neg q)$	$(p \vee q)$	$(\neg(p \vee q))$	$((\neg p) \wedge (\neg q))$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Since the final two columns are the same, the formulas are logically equivalent. $\square$

(6)[10 pts] Use a truth table to determine whether $(p \vee (q \wedge r))$ and $((p \vee q) \wedge (p \vee r))$ are logically equivalent.

Solution.

p	q	r	$(q \wedge r)$	$(p \vee q)$	$(p \vee r)$	$(p \vee (q \wedge r))$	$((p \vee q) \wedge (p \vee r))$
T	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	F	T	F	F
T	T	F	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	F	F	T	F	F	F
F	F	F	F	F	F	F	F

Since the final two columns are the same, the formulas are logically equivalent. $\square$

(7) Let p be "It is raining", q be "The parade is canceled" and r be "The streets are flooded". Translate the following formulas into English (don't forget to end sentences with a period).

(a)[3 pts] $((\neg q) \rightarrow (\neg p))$

Solution. If the parade isn't canceled, then it is not raining. □

(b)[3 pts] $((p \wedge r) \rightarrow q)$

Solution. If it is raining and the streets are flooded, then the parade is canceled. □

(c)[4 pts] $((q \vee r) \leftrightarrow p)$

Solution. The parade is canceled or the streets are flooded if and only if it is raining. □

(8) Let p be "John is injured", q be "John plays", and r be "John's team loses the game". Translate the following sentences into formulas. I want proper syntax here (so be careful not to omit parentheses). In what follows, assume that the game is either won or lost (that is, ties are impossible).

(a)[3 pts] If John is injured or John's team loses the game, then John didn't play.

Solution. $((p \vee r) \rightarrow (\neg q))$. □

(b)[3 pts] John is not injured, John plays, or John's team wins the game.

Solution. $((\neg p) \vee (q \vee (\neg r)))$. □

(c)[4 pts] John's team wins the game if and only if John is both injured and does not play.

Solution. $((\neg r) \leftrightarrow (p \wedge (\neg q)))$. □

(9) Negate (in English) the following using Demorgan's Laws (see notes for examples). You can simply write the negation without supporting work if you'd like.

(a)[3 pts] It is Monday and it is cloudy.

Solution. It is not Monday or it is not cloudy. □

(b)[3 pts] Jane is tall or Jane is sick.

Solution. Jane is not tall and Jane is not sick. □

(c)[4 pts] John is happy or John is both sad and angry. (you will need to perform two negations)

Solution. John is not happy and either John is not sad or John is not angry. □