**Math 2150 Assignment 3 Solutions**

(1) Let $P(x)$ be the one-place predicate "$x + 5$ is an integer", and let $Q(z)$ be the predicate "$z$ has 6 legs".

(a)[2 pts] What is $P(1)$?

*Solution.* $P(1)$ is "$1 + 5$ is an integer." $\square$

(b)[2 pts] What is $P(y)$?

*Solution.* $P(y)$ is "$y + 5$ is an integer." $\square$

(c)[2 pts] What is $Q(John)$?

*Solution.* $Q(John)$ is "John has 6 legs." $\square$

(2) Let $P(x)$ be "$x$ has a job", and let the domain for $x$ be the collection of all students. Translate to English (do this the "geeky way" that I outlined in the notes, remember to use the phrase "such that" when appropriate, say what kind of object $x$ is, etc.).

(a)[3 pts] $\forall x P(x)$

*Solution.* For every student $x$, $x$ has a job. $\square$

(b)[3 pts] $\exists x P(x)$

*Solution.* There exists a student $x$ such that $x$ has a job. $\square$

(c)[4 pts] $\forall x(P(x) \vee (\neg P(x)))$

*Solution.* For every student $x$, $x$ has a job or $x$ does not have a job. $\square$

(3) Let $P(y)$ be "$y$ is even". Do the following (no justification required):

(a)[3 pts] Find a domain for $y$ so that $\forall y P(y)$ is true.

*Solution.* Let the domain be $\{2\}$. $\square$

(b)[3 pts] Find a domain for $y$ so that $\forall y P(y)$ is false.

*Solution.* Let the domain be the set of all integers. $\square$

(c)[3 pts] Find a domain for $y$ so that $\exists y P(y)$ is false.

*Solution.* Let the domain be the set of all odd integers. $\square$

(4) Using the notes (see relevant examples I did), justify why the following are formulas:

(a)[6 pts] $(\exists x(\neg P(x)) \leftrightarrow (Q(z) \vee F(y)))$

*Solution.* $F(y)$ and $Q(z)$ are formulas by 1. Now $(Q(z) \vee F(y))$ is a formula by 2. $P(x)$ is a formula by 1. Now by 2., $(\neg P(x))$ is a formula. By 3., $\exists x(\neg P(x))$ is a formula. Finally, by 2., $(\exists x(\neg P(x)) \leftrightarrow (Q(x) \vee F(y)))$ is a formula. □

(b)[6 pts] $\forall x(P(x) \leftrightarrow \exists y(Q(y) \wedge A(y)))$

*Solution.* By 1., both $A(y)$ and $Q(y)$ are formulas. Now by 2., $(Q(y) \wedge A(y))$ is a formula. By 3., $\exists y(Q(y) \wedge A(y))$ is a formula. By 1., $P(x)$ is a formula. By 2., $(P(x) \leftrightarrow \exists y(Q(y) \wedge A(y)))$ is a formuls. Finally, by 3., $\forall x(P(x) \leftrightarrow \exists y(Q(y) \wedge A(y)))$ is a formula. □

(5) Use the recursive definition of "freeness" to justify the following (see examples in notes). You do NOT need to justify why particular subformulas actually are subformulas, so please don't confuse this with those kinds of problems from (4).

(a)[4 pts] $z$ occurs free in the formula $\exists y P(z)$ (here $y$ and $z$ are different variables, of course).

*Solution.* $z$ occurs free in $P(z)$ by 1. Now by 4., $z$ occurs free in $\exists y P(z)$. □

(b)[4 pts] $y$ occurs free in $((R(x) \to \forall x P(x)) \leftrightarrow Q(y))$.

*Solution.* $y$ occurs free in $Q(y)$ by 1. Now by 3., $y$ occurs free in $((R(x) \to \forall x P(x)) \leftrightarrow Q(y))$. □

(c)[4 pts] $z$ occurs free in $((\neg P(z)) \leftrightarrow \exists x Q(x))$

*Solution.* $z$ occurs free in $P(z)$ by 1. and now by 2., $z$ occurs free in $(\neg P(z))$. By 3, $z$ occurs free in $((\neg P(x)) \leftrightarrow \exists x Q(x))$. □

(d)[4 pts] $x$ occurs free in $(\forall x(R(x) \vee P(x)) \vee S(x))$. (If you think $x$ doesn't occur free, try building up the formula from the predicates using the recursive definition (but you need not show me this work))

*Solution.* $x$ occurs free in $S(x)$ by 1., and now by 3., $x$ occurs free in $(\forall x(R(x) \vee P(x)) \vee S(x))$. □

(6) Let $P(x)$ be "$x$ is positive", $Q(x)$ be "$x$ is negative", $R(x)$ be "$x$ is an integer". Translate the following into formulas, where the domain for $x$ is the collection of all real numbers. USE CORRECT SYNTAX.

(a)[4 pts] There exists a real number which is positive but not an integer.

*Solution.* $\exists x(P(x) \land (\neg R(x)))$. □

(b)[4 pts] Every real number that is positive is an integer.

*Solution.* $\forall x(P(x) \to R(x))$. □

(c)[4 pts] Every real number is both positive and negative.

*Solution.* $\forall x(P(x) \land Q(x))$. □

(d)[4 pts] A real number is positive if and only if it is a negative integer.

*Proof.* $\forall x(P(x) \leftrightarrow (Q(x) \land R(x)))$. □

(7) Let $P(x)$ be "$x$ is a student in Math 4140" and $Q(x)$ be "$x$ is an engineering major".

(a)[3 pts] Express "Every Math 4140 student is not an engineering major" as a formula if the domain for $x$ is the collection of students in Math 4140. (I do a problem VERY similar to this in the notes)

*Solution.* $\forall x(\neg Q(x))$. □

(b)[3 pts] Express "Every Math 4140 student is an engineering major" as a formula if the domain for $x$ is the collection of all UCCS students.

*Solution.* $\forall x(P(x) \to Q(x))$. □

(c)[3 pts] Express "There exists a Math 4140 student that is an engineering major" if the domain for $x$ is the collection of students in Math 4140.

*Solution.* $\exists x Q(x)$. □

(d)[3 pts] Express "There exists a Math 4140 student that is an engineering major" if the domain for $x$ is the collection of all UCCS students.

*Solution.* $\exists x(P(x) \land Q(x))$. □