Multirate Sampling Simulation Using MATLAB's Signal Processing Toolbox

Introduction

This technical note explains how you can very easily use the command line functions available in the MATLAB signal processing toolbox, to simulate simple multirate DSP systems. The focus here is to be able to view in the frequency domain what is happening at each stage of a system involving upsamplers, downsamplers, and lowpass filters. All computations will be performed using MATLAB and the signal processing toolbox. These same building blocks are available in Simulink via the DSP blockset. The DSP blockset allows better visualization of the overall system, but is not available in the ECE general computing laboratory or on most personal systems. A DSP block set example will be included here just so one can see the possibilities with the additional MATLAB tools.

Command Line Building Blocks

To be able to visualize the spectra in a multirate system we need the basic building blocks of

- Bandlimited signal generation; here we create a signal with a triangle shaped spectrum using `sinc()`
- Integer upsampling and downsampling operations; here we use the signal processing toolbox functions `upsample()` and `downsample()`
- Lowpass filtering for antialiasing and interpolation; here we use `fir1()`
- If we don't care to examine the spectra between the antialiasing lowpass filter and the decimator or between the upsampler and interpolation filter, we can use combination functions such as `decimate()`, `interpolate()`, and `resample()`

Signal Generation

In a theoretical discussion of sampling theory it is common place to represent the signal of interest as having a triangular shaped Fourier spectrum. Another common signal type is one or more discrete-time sinusoids. We know that

$$\frac{\sin \omega_c n}{\pi n} \overset{\mathcal{F}}{\leftrightarrow} \prod\left(\frac{\omega}{2\omega_c}\right) \tag{1}$$

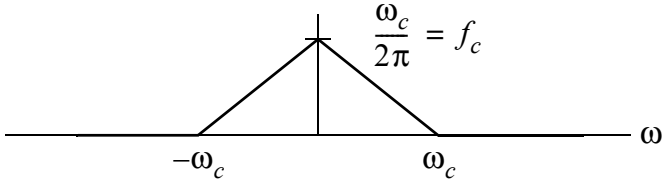
where

$$\prod\left(\frac{x}{W}\right) = \begin{cases} 1 & -W/2 < x < W/2 \\ 0 & \text{otherwise} \end{cases}$$

A triangle can be obtained in the frequency domain by noting that

$$\left[\frac{\sin(\omega_c n/2)}{\pi n}\right]^2 \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2\pi}\int_{-\pi}^{\pi} \prod\left(\frac{\omega-\theta}{\omega_c}\right)\prod\left(\frac{\theta}{\omega_c}\right)d\theta \tag{2}$$

The periodic convolution on the right-side of (2) yields

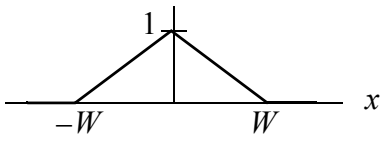


where f_c is the spectral bandwidth (single-sided or lowpass) in normalized frequency units. It then follows that for a unit height spectrum we have the transform pair

$$\frac{2\pi}{\omega_c}\left(\frac{\sin(\omega_c n/2)}{\pi n}\right)^2 \overset{\mathcal{F}}{\leftrightarrow} \Lambda\left(\frac{\omega}{\omega_c}\right) \tag{3}$$

where

$$\Lambda\left(\frac{x}{W}\right) \quad =$$



Using the sinc( ) function in MATLAB, which is defined as

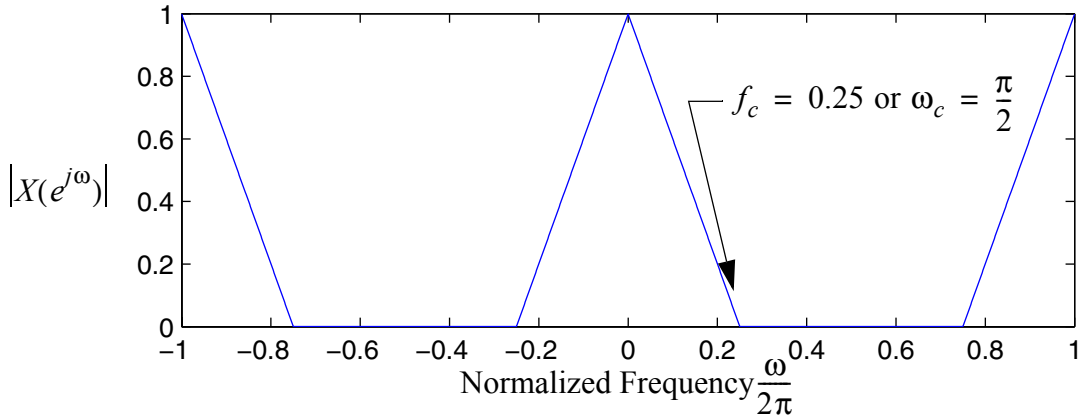
$$\text{sinc}(x) \equiv \frac{\sin \pi x}{\pi x} \tag{4}$$

we can write (3) as

$$f_c[\text{sinc}(f_c n)]^2 \overset{\mathcal{F}}{\leftrightarrow} \Lambda\left(\frac{\omega}{2\pi f_c}\right) \tag{5}$$

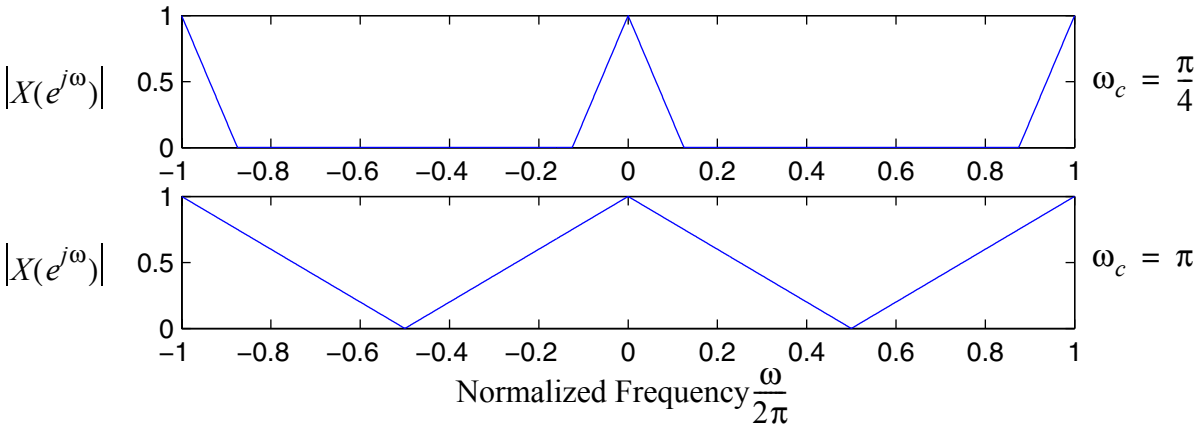
Creating a triangular spectrum signal in MATLAB just requires delaying the signal in samples so that both tails can be represented in a causal simulation, e.g.,

```
>> n = 0:1024;
>> x = 1/4*sinc(1/4*(n-512)).^2; % set peak of signal to center of interval
>> f = -1:1/512:1; % create a custom frequency axis for spectral plotting
>> X = freqz(x,1,2*pi*f); %Compute the Fourier transform of x
>> plot(f,abs(X))
>> print -tiff -depsc multi1.eps
```



Consider a couple of more examples:

```
>> n = 0:1024;
>> x125 = 1/8*sinc(1/8*(n-512)).^2;
>> x5 = 1/2*sinc(1/2*(n-512)).^2;
>> f = -1:1/512:1;
>> X125 = freqz(x125,1,2*pi*f);
>> X5 = freqz(x5,1,2*pi*f);
>> subplot(211)
>> plot(f,abs(X125))
>> subplot(212)
>> plot(f,abs(X5))
>> print -tiff -depsc multi2.eps
```



Upsample and Downsample

With a means to generate a signal having bandlimited spectra in place, we can move on to the upsampling and downsampling operations. The signal processing toolbox has dedicated functions for doing this operation, although they are actually quite easy to write yourself.

```
>> help downsample
```

```
 DOWNSAMPLE Downsample input signal.
    DOWNSAMPLE(X,N) downsamples input signal X by keeping every
    N-th sample starting with the first. If X is a matrix, the
    downsampling is done along the columns of X.

    DOWNSAMPLE(X,N,PHASE) specifies an optional sample offset.
    PHASE must be an integer in the range [0, N-1].

    See also UPSAMPLE, UPFIRDN, INTERP, DECIMATE, RESAMPLE.

>> help upsample

 UPSAMPLE Upsample input signal.
    UPSAMPLE(X,N) upsamples input signal X by inserting
    N-1 zeros between input samples.  X may be a vector
    or a signal matrix (one signal per column).

    UPSAMPLE(X,N,PHASE) specifies an optional sample offset.
    PHASE must be an integer in the range [0, N-1].

    See also DOWNSAMPLE, UPFIRDN, INTERP, DECIMATE, RESAMPLE.
```

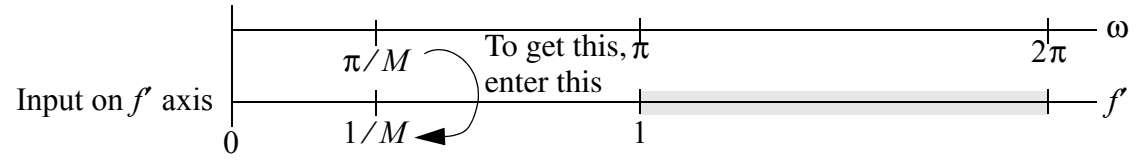
These functions will be used in examples that follow.

Filtering

To implement a simple yet effective lowpass filter to prevent aliasing in a downsampler and interpolation in an upsampler, we can use the function `fir1()` which designs linear phase FIR filters using a windowed sinc function.

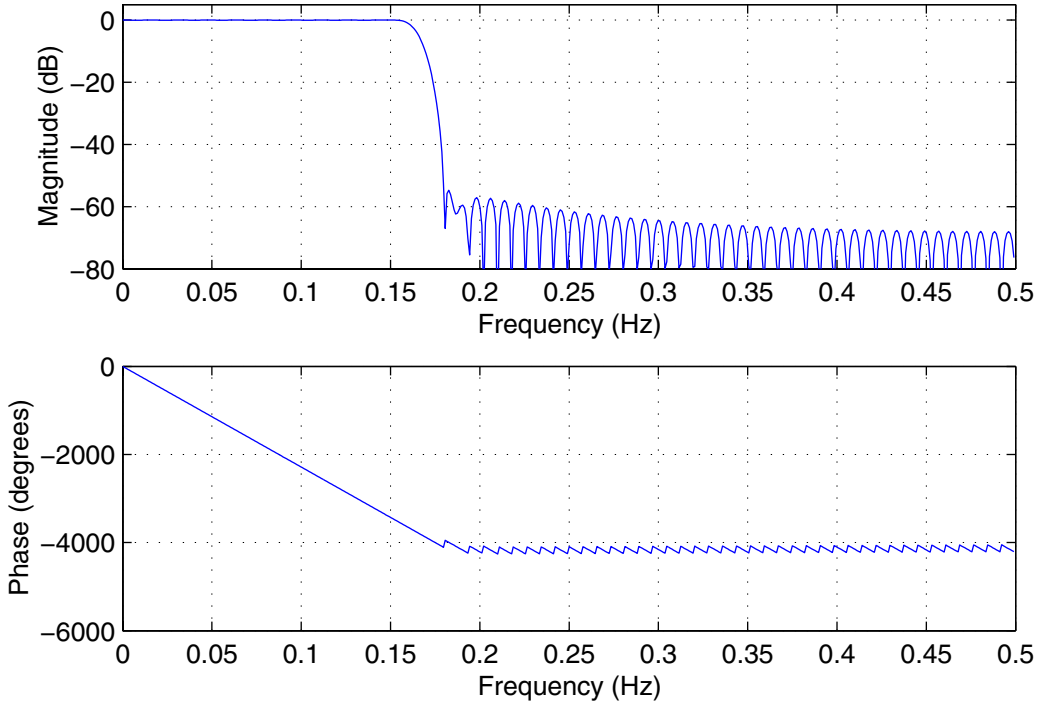
```
>> help fir1

 FIR1   FIR filter design using the window method.
    B = FIR1(N,Wn) designs an N'th order lowpass FIR digital filter
    and returns the filter coefficients in length N+1 vector B.
    The cut-off frequency Wn must be between 0 < Wn < 1.0, with 1.0
    corresponding to half the sample rate.  The filter B is real and
    has linear phase.  The normalized gain of the filter at Wn is
    -6 dB.

    More help beyond this, but this is the basic LPF design interface
```

The filter design functions use half sample rate normalized frequencies:



To design a lowpass filter FIR filter having 128 coefficients and a cutoff frequency of $\omega_c = \pi/3$, we simply type

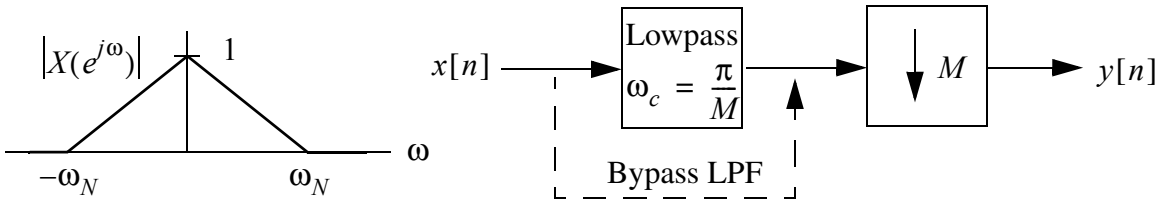
```
>> h = fir1(128-1,1/3); % Input cutoff as 2*fc, where fc = wc/(2pi)
>> freqz(h,1,512,1)
>> print -tiff -depsc multi3.eps
```



System Simulation

To keep things simple we will consider just simple decimator and interpolator systems.

A Simple Decimation Example

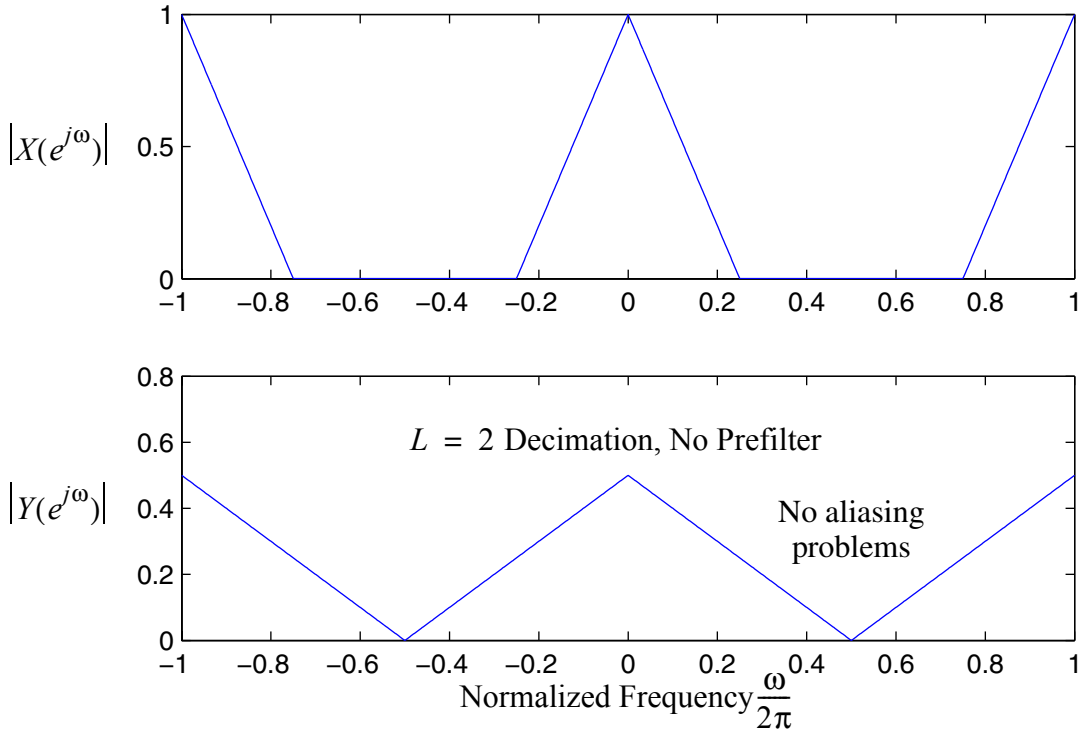


In the above system we will consider the pure decimator and the decimator with lowpass prefiltering. Two different values of M will be considered.

As a first example we pass directly into an $M = 2$ decimator with a signal having $\omega_N = \pi/2$ ($f_N = 1/4$).

```
>> n = 0:1024;
```

```
>> x = 1/4*sinc(1/4*(n-512)).^2;
>> y = downsample(x,2);
>> f = -1:1/512:1;
>> X = freqz(x,1,2*pi*f);
>> Y = freqz(y,1,2*pi*f);
>> plot(f,abs(X))
>> subplot(211)
>> plot(f,abs(X))
>> subplot(212)
>> plot(f,abs(Y))
>> print -tiff -depsc multi4.eps
```

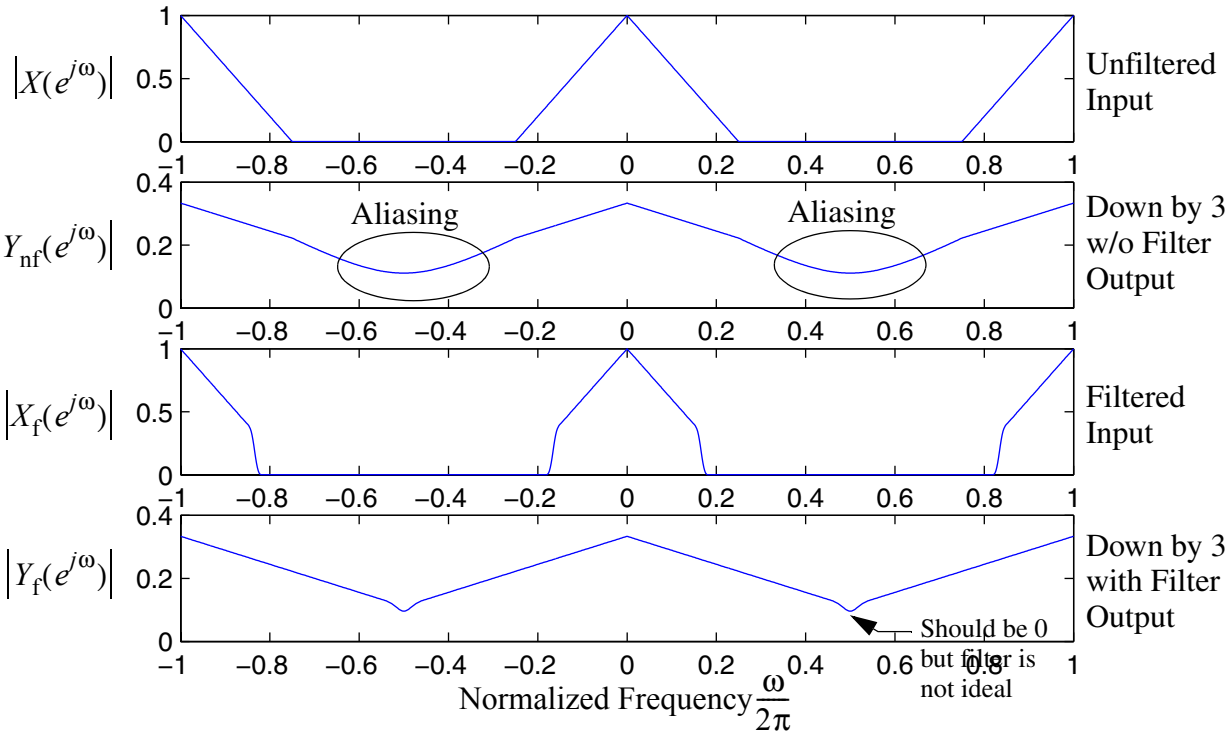


- The results are exactly as we would expect

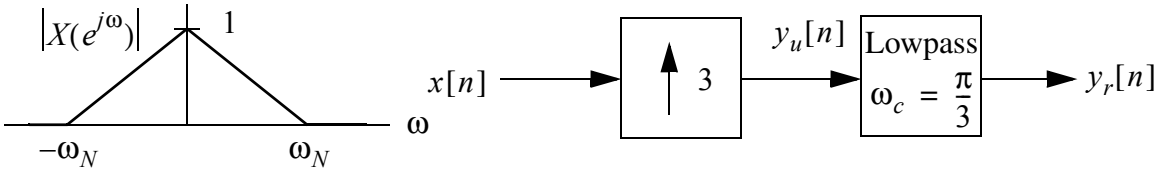
Now, use the same signal except with $M = 3$. First without the prefilter, and then include it to avoid aliasing.

```
>> n = 0:1024;
>> x = 1/4*sinc(1/4*(n-512)).^2;
>> xf = filter(h,1,x);
>> yf = downsample(xf,3);
>> ynf = downsample(x,3);
>> X = freqz(x,1,2*pi*f);
>> Xf = freqz(xf,1,2*pi*f);
>> Yf = freqz(yf,1,2*pi*f);
>> Ynf = freqz(ynf,1,2*pi*f);
>> subplot(411)
```

```
>> plot(f,abs(X))
>> subplot(412)
>> plot(f,abs(Ynf))
>> subplot(413)
>> plot(f,abs(Xf))
>> subplot(414)
>> plot(f,abs(Yf))
>> print -tiff -depsc multi5.eps
```



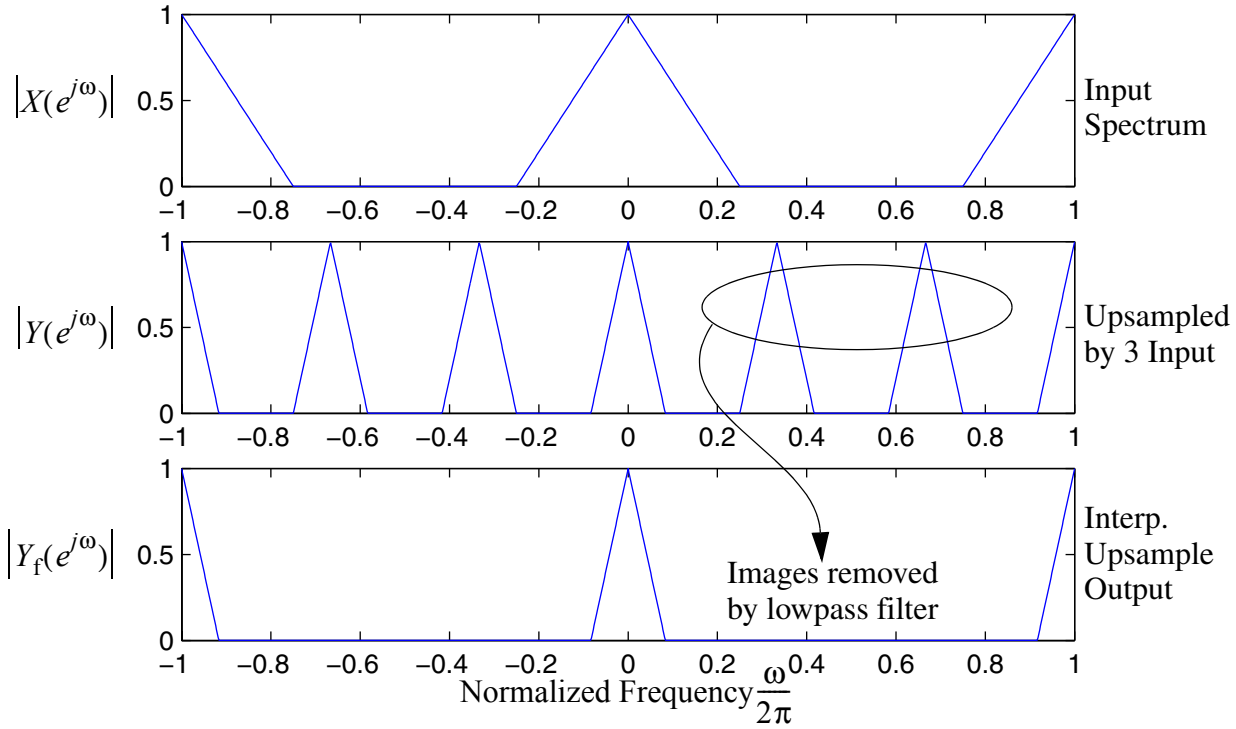
A Simple Interpolation System



In the above system we will consider the pure upsampler and the upsampler followed by a low-pass interpolation filter. Let $L = 3$ and the signal have $\omega_N = \pi/2$ ($f_N = 1/4$).

```
>> n = 0:1024;
```

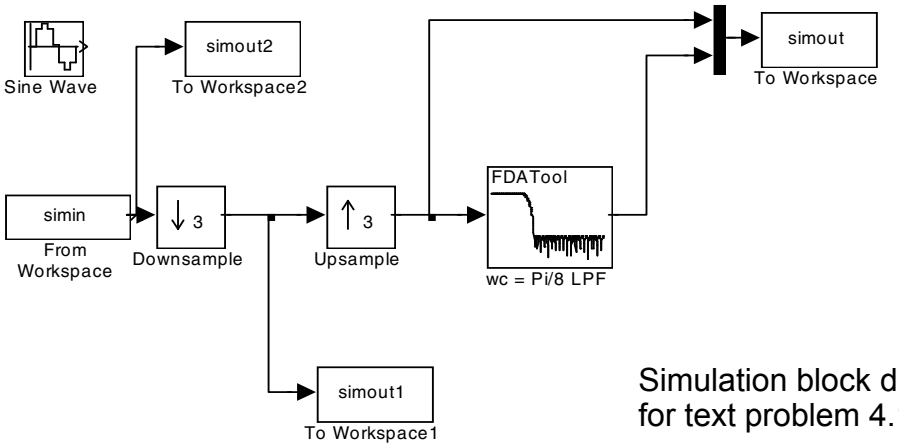
```
>> x = 1/4*sinc(1/4*(n-512)).^2;
>> y = upsample(x,3);
>> X = freqz(x,1,2*pi*f);
>> Y = freqz(y,1,2*pi*f);
>> h = fir1(128-1, 1/3);
>> yf = filter(h,1,y);
>> Yf = freqz(yf,1,2*pi*f);
>> subplot(311)
>> plot(f,abs(X))
>> subplot(312)
>> plot(f,abs(Y))
>> subplot(313)
>> plot(f,abs(Yf))
>> print -tiff -depsc multi6.eps
```



A DSP Blockset Example

What can be accomplished at the MATLAB command line using just the signal processing toolbox, can be enhanced significantly by adding Simulink and the DSP blockset. Simulink is a block diagram based simulation environment that sits on top of MATLAB. The DSP blockset augments Simulink with a DSP specific block library and requires that the signal processing toolbox be present.

As a simple example consider Text problem 4.15. The results are not shown here as they are in



Simulation block diagram for text problem 4.15

the solutions to problem 4.15. A sinusoidal input can be connected (the plot currently off to the side) or a signal vector from the MATLAB workspace can be used as the input. As shown above the input from the workspace is a triangular spectrum signal. The upsampler and downsampler blocks works just like the command line functions. The lowpass filter block has been designed using a GUI filter design tool (FDA tool), but ultimately uses coefficients similar to those obtained from MATLAB's command line filter design tools. The To Workspace blocks allow the signals to be exported to the MATLAB workspace for futher manipulation.