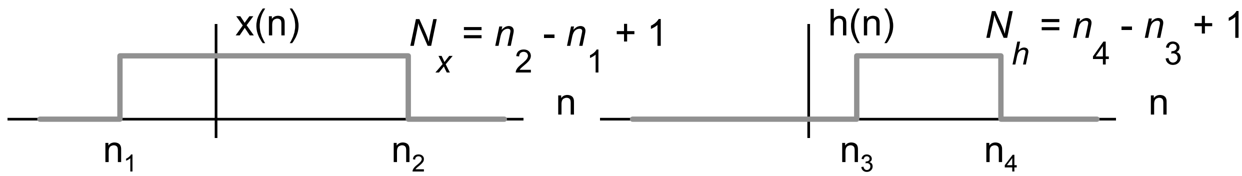
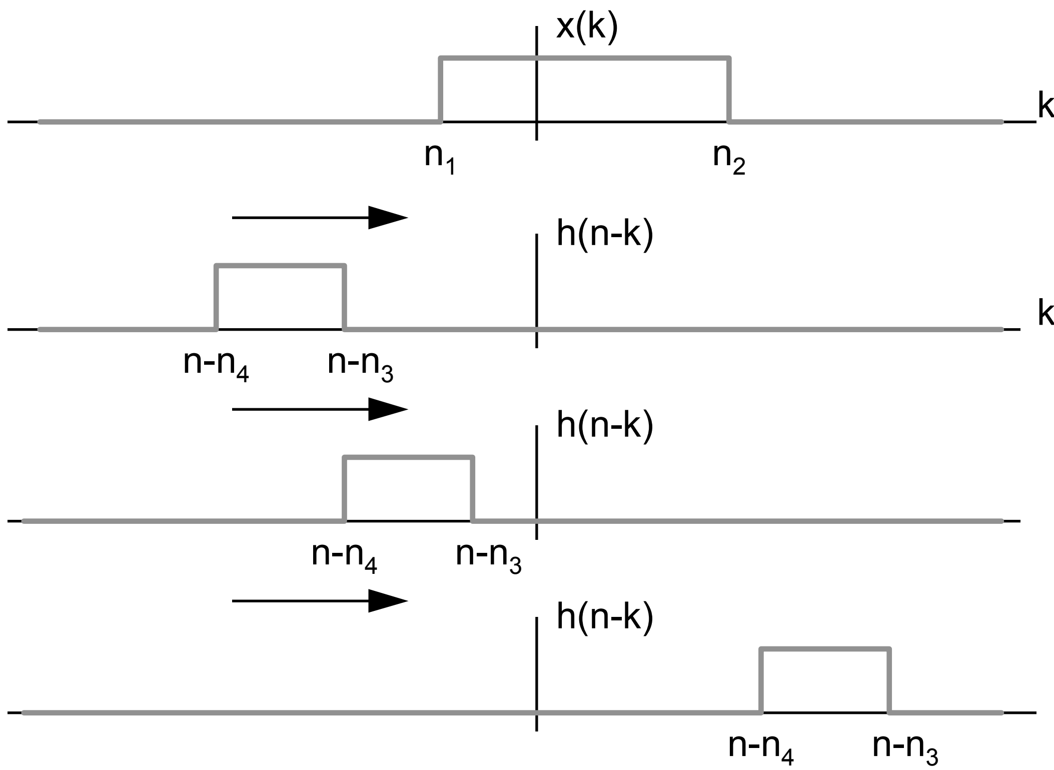
Example: Consider two finite length sequences



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- The sum must be evaluated for each value of n



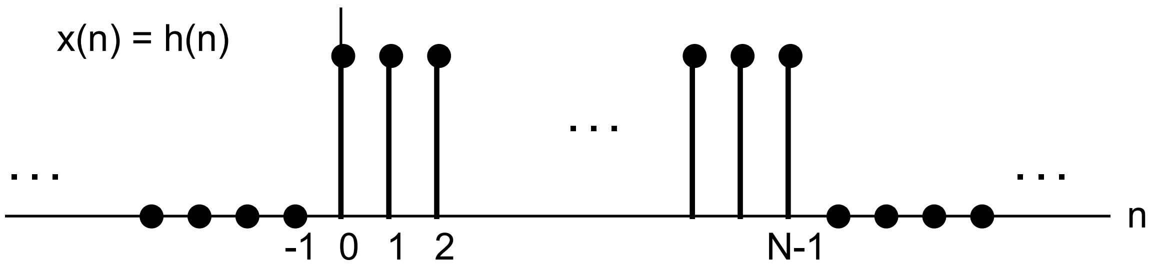
- The first nonzero value occurs when $n - n_3 = n_1$ or $n = n_1 + n_3$
- The last nonzero value occurs when $n - n_4 = n_2$ or $n = n_2 + n_4$

– Summary: $y(n)$ is nonzero for at most

$$n_1 + n_3 \le n \le n_2 + n_4 \;\Rightarrow\; N_y = N_x + N_h - 1$$

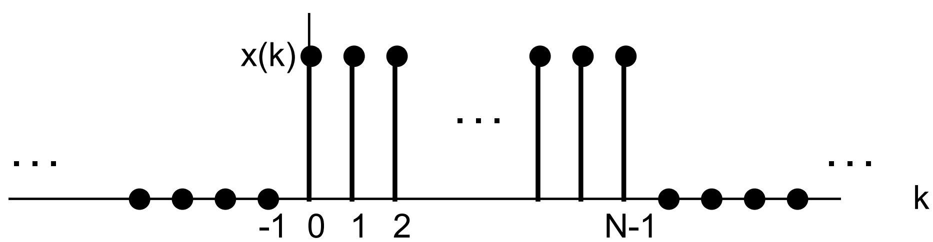
- Example: Two finite duration sequences given by

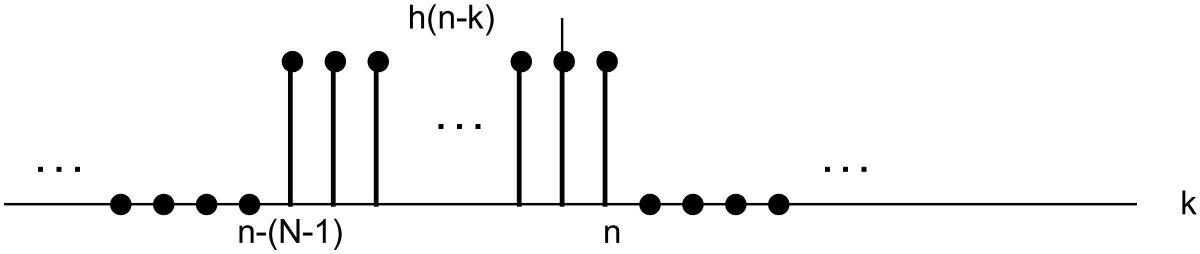
$$x[n] = h[n] = u[n] - u[n - N],\ N > 0$$



– From the previous example we know that $y[n]$ will be at most nonzero on the interval $[0, 2(N-1)]$

– There are 4 cases to consider:





– *Case 1*: $n < 0$ which implies no overlap, so $y[n] = 0$

– *Case 2*: $0 \le n \le N - 1$

$$y[n] = \sum_{k=0}^{n} (1)(1) = n + 1$$

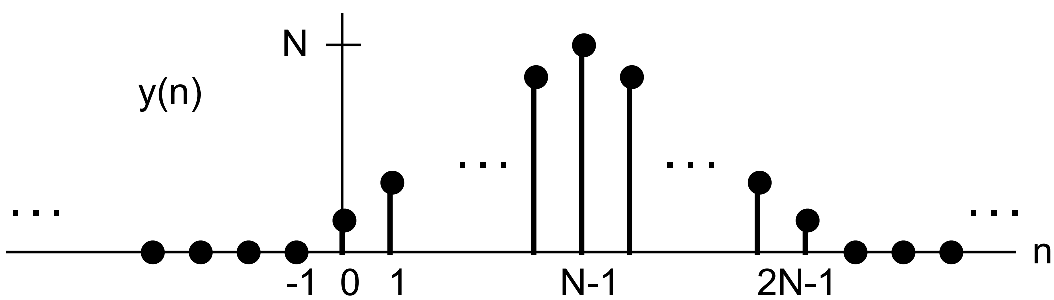
– *Case3*: $N-1 \leq n \leq 2N-2$

$$y[n] = \sum_{k=n-(N-1)}^{N-1} (1)(1) = (N-1) - \big(n(N-1)\big) + 1$$

– *Case 4*: $n > 2N-2$ which implies no overlap, so $y[n] = 0$

– In summary:

$$y[n] = \begin{cases} n+1, & 0 \leq n \leq N-1 \\ 2N-1-n, & N-1 \leq n \leq 2N-2 \\ 0, & \text{otherwise} \end{cases}$$



<u>Example</u>: Two finite duration sequences in sequence explicit representation:

$$h[n] = \{1,\underset{\uparrow}{2},1,-1\}, \quad x[n] = \{\underset{\uparrow}{1},2,3,1\}$$

– In the above notation the arrows indicate where $n = 0$

– We need to evaluate the convolution sum for $-1 \le n \le 5$

– To evaluate construct the following table:

				$x[k]$ vs k							
$h[n-k]$		0	0	1	2	3	1	0	0		$y[n]$
n = -1	-1	1	2	1	0	0	0	0	0	0	1
n = 0	0	-1	1	2	1	0	0	0	0	0	4
n = 1	0	0	-1	1	2	1	0	0	0	0	8
n = 2	0	0	0	-1	1	2	1	0	0	0	8
n = 3	0	0	0	0	-1	1	2	1	0	0	3
n = 4	0	0	0	0	0	-1	1	2	1	0	-2
n = 5	0	0	0	0	0	0	-1	1	2	1	-1

– The final output is thus

$$y[n] = \{\ldots 0,1,\underset{\uparrow}{4},8,8,3,-2,-1,0,0,\ldots\}$$

– Is this reasonable? The output should start at (-1 + 0) = -1 and should stop at (3 + 2) = 5, which is indeed the case

<u>Example</u>: Two infinite duration sequences,

$$x[n] = u[n], \quad h[n] = a^n u[n]$$

– By direct substitution into the convolution sum formula we have

$$y[n] = \sum_{k=-\infty}^{\infty} a^k u[k] u[n-k]$$

– The term $u(k)$ sets the lower sum limit to zero while the term $u(n - k)$ sets the upper sum limit to n, hence

$$y[n] = \begin{cases} 0, & n < 0 \\ \sum_{k=0}^{n} a^k, & n \geq 0 \end{cases} = \frac{1 - a^{n+1}}{1-a} u[n]$$