Impulse Train Spectra Fall 2024

```
# %pylab inline
from numpy import *
from matplotlib.pyplot import *
%matplotlib inline
import sk_dsp_comm.sigsys as ss
import scipy.signal as signal
from IPython.display import Audio, display
from IPython.display import Image, SVG

%config InlineBackend.figure_formats=['svg'] # SVG inline viewing
```

Spectrum of an Impulse Train

- The theory says a period M discrete-time pulse train has Fourier transform pair:

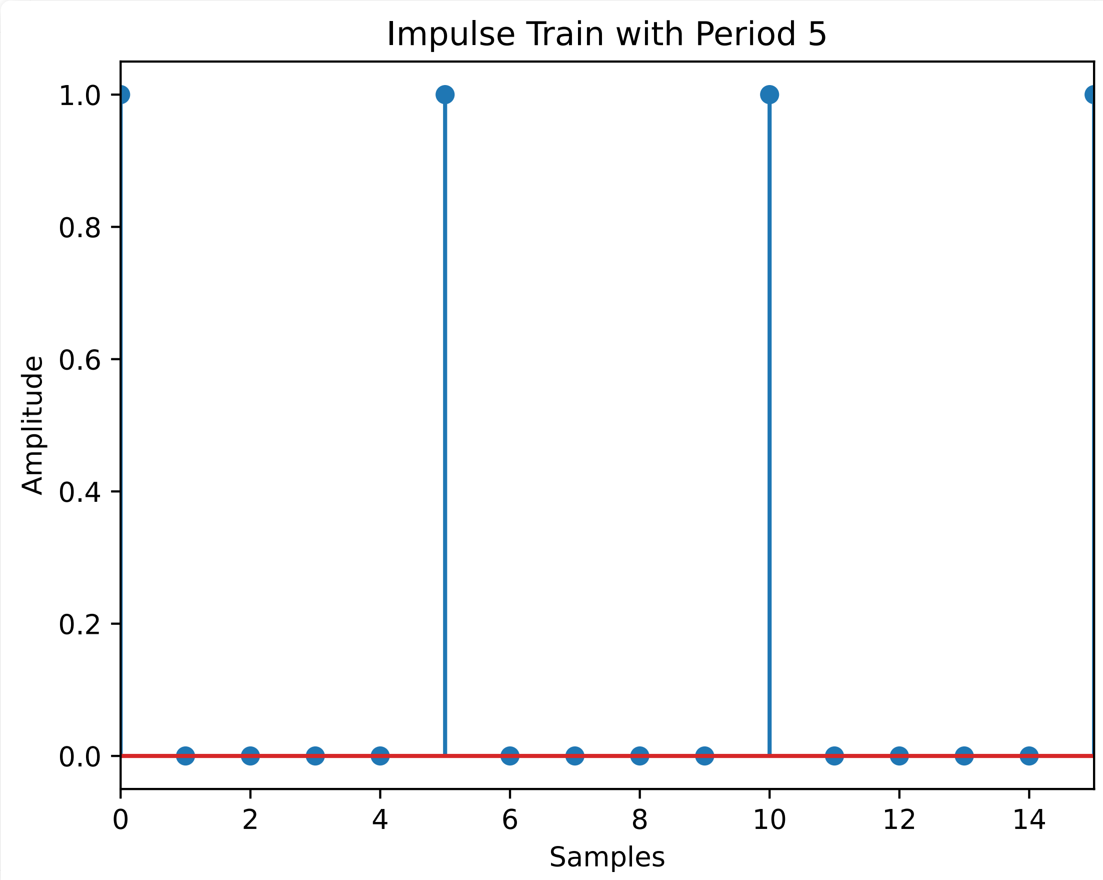
$$\sum_{k=-\infty}^{\infty} \delta[n - M\,k] \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \frac{2\pi}{M} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi}{M} n\right) \tag{1}$$

Modeling in Python

- To generate an impulse train we start with `ones(100)` and then up sample the sequency by a factor of 5. This will create an impulse train haing 4 zero sample stuffed every 1 in `x`.

```
x = ones(100)
y = ss.upsample(x,5) # we learn about upsampling in Chapter 4
stem(y[0:100])
title(r'Impulse Train with Period 5')
ylabel(r'Amplitude')
xlabel(r'Samples')
xlim([0,15])
len(y)
```

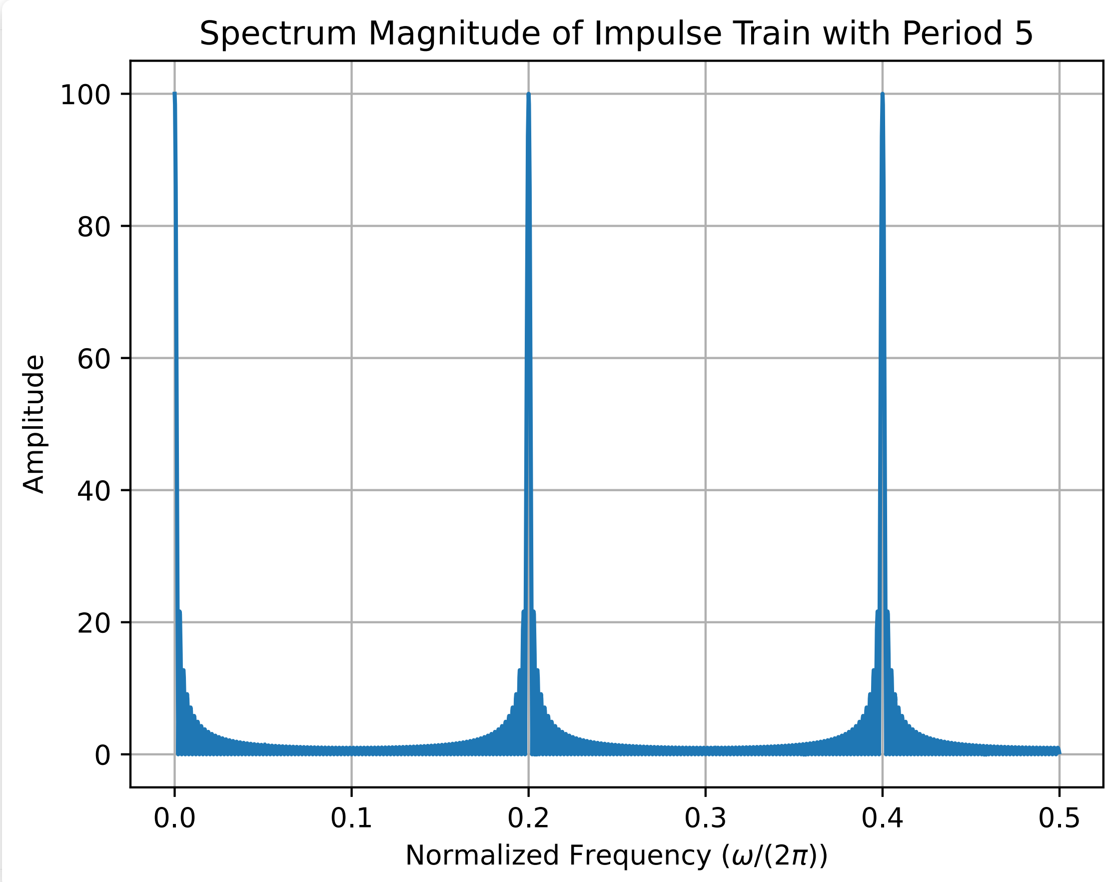
```
500
```



- Next we approximate the Fourier transform of `y` using `freqz()` by loading `y` as the vector of coefficients `b` in `freqz`
- In the signal model $y[n]$ is doubly infinite, but here we Fourier transforming using $y[n], 0 \leq n \leq 500 - 1$

```
f = arange(0,.5,1/5000)
w,Y = signal.freqz(y,1,2*pi*f)
```

```
plot(f,abs(Y))
title(r'Spectrum Magnitude of Impulse Train with Period 5')
ylabel(r'Amplitude')
xlabel(r'Normalized Frequency ($\omega/(2\pi)$)')
grid();
```



- The spectral spikes are not true delta functions with a scaled amplitude and a finite width with sidelobes
- The spacing is as expected, $\Delta f = 1/5$ or $\Delta\omega = 2\pi/5$

Note: To produce this document from Markdown I used a nicer, *for-pay*, Markdown editor: https://typora.io. I used this editor to produce all of the lab readers for ECE 4670 - Communications Lab.