ECE 5650/4650 Tables for Exam I

Sequence Sum Formulas

Sum	Condition
$\sum_{k=0}^{n} k = \frac{1}{2}n(n+1)$	none
$\sum_{k=N_1}^{N_2} k = \frac{1}{2}\left[(N_2^2 + N_2) - (N_1 + (N_1 - 1)^2 - 1)\right]$	$N_1 < N_2$
$\sum_{k=N_1}^{N_2} a^k = \frac{a^{N_1} - a^{N_2+1}}{1-a}$	none
$\sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}$	none
$\sum_{k=0}^{n} ka^k = \frac{a\left[1-(n+1)a^n + na^{n+1}\right]}{(1-a)^2}$	none
$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$	$\|a\| < 1$
$\sum_{k=0}^{\infty} ka^k = \frac{a}{(1-a)^2}$	$\|a\| < 1$

Fourier Transform Symmetry Properties

Sequence $x[n]$	Fourier Transform $X(e^{j\omega})$
1. $x^*[n]$	$X^*(e^{-j\omega})$
2. $x^*[-n]$	$X^*(e^{j\omega})$
3. $\text{Re}\{x[n]\}$	$X_e(e^{j\omega})$ (conjugate symmetric part of $X(e^{j\omega})$)
4. $\text{Im}\{x[n]\}$	$X_o(e^{j\omega})$ (conjugate anti - symmetric part of $X(e^{j\omega})$)
5. $x_e[n]$	$X_R(e^{j\omega})$
6. $x_o[n]$	$jX_I(e^{j\omega})$
Real $x[n]$ only	
7. Any real $x[n]$	$X(e^{j\omega}) = X^*(e^{-j\omega})$ (FT is conjuugate symmetric)
8. Any real $x[n]$	$X_R(e^{j\omega}) = X_R(e^{-j\omega})$ (real part is even)
9. Any real $x[n]$	$X_I(e^{j\omega}) = -X_I(e^{-j\omega})$ (imaginary part is odd)
10. Any real $x[n]$	$\left\|X(e^{j\omega})\right\| = \left\|X(e^{-j\omega})\right\|$ (magnitude is even)
11. Any real $x[n]$	$\angle X(e^{j\omega}) = -\angle X(e^{-j\omega})$
12. $x_e[n]$ (even part)	$X_R(e^{j\omega})$
13. $x_o[n]$ (odd part)	$jX_I(e^{j\omega})$

Summary of Fourier Transform Theorems

Sequence	Fourier Transform
$x[n]$, $y[n]$	$X(e^{j\omega})$, $Y(e^{j\omega})$
1. $ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
2. $x[n - n_d]$ (n_d an integer)	$e^{-j\omega n_d} X(e^{j\omega})$
3. $e^{j\omega_0 n} x[n]$	$X(e^{j(\omega-\omega_0)})$
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real
5. $nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$
6. $x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
7. $x[n]y[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
Parsevals Theorem	
8. $\sum_{n=-\infty}^{\infty} \lvert x[n] \rvert^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left\lvert X(e^{j\omega}) \right\rvert^2 d\omega$	
9. $\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$	

Useful Fourier Transform Pairs

Sequence	Fourier Transform
1. $\delta[n]$	1
2. $\delta[n - n_0]$	$e^{-j\omega n_0}$
3. $1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
4. $a^n u[n] \quad (\lvert a \rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
5. $u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
6. $(n + 1)a^n u[n]$	$\frac{1}{(1-ae^{-j\omega})^2}$
7. $\frac{r^n \sin \omega_p (n+1)}{\sin \omega_p} u[n] \; (\lvert r \rvert < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega}+r^2 e^{-j2\omega}}$
8. $\frac{\sin \omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \omega_c < \lvert\omega\rvert < \pi \end{cases}$
9. $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
10. $e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
11. $\cos(\omega_0 n + \phi)$	$\pi \sum_{k=-\infty}^{\infty} \left[e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)\right]$

A Summary of z-Transform Theorems

Sequence	Transform	ROC
1. $ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$R_{x_1} \cap R_{x_2}$
2. $x[n - n_0]$	$z^{-n_0}X(z)$	R_x, except possibly zero or infinity
3. $z_0^n x[n]$	$X(z/z_0)$	$\|z_0\|\, R_x$
4. $nx[n]$	$-z\frac{dX(z)}{dz}$	R_x, except possibly zero or infinity
5. $x^*[n]$	$X^*(z^*)$	R_x
6. $\mathrm{Re}\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	Contains R_x
7. $\mathrm{Im}\{x[n]\}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	Contains R_x
8. $x[-n]$	$X(1/z)$	$1/R_x$
9. $x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x_1} \cap R_{x_2}$
10. Initial value theorem: $x[n] = 0,\ n < 0$	$\lim_{z\to\infty} X(z) = x[0]$	

Useful z-Transform Pairs

$x[n]$	$X(z)$	ROC
$\delta[n]$	1	all z
$\delta[n-m]$	z^{-m}	all z except $z = 0$ if $m > 0$ and $\|z\| = \infty$ if $m < 0$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
$(n+1)a^n u[n]$	$\frac{1}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$\frac{(n+1)(n+2)\cdots(n+k-1)}{(k-1)!}a^n u[n]$	$\frac{1}{(1-az^{-1})^k}$	$\|z\| > \|a\|$
$r^n \cos(\omega_o n)u[n]$	$\frac{1-(r\cos\omega_o)z^{-1}}{1-(2r\cos\omega_o)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
$r^n \sin(\omega_o n)u[n]$	$\frac{(r\sin\omega_o)z^{-1}}{1-(2r\cos\omega_o)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
$\cosh(nb)u[n]$	$\frac{1-(\cosh b)z^{-1}}{1-(2\cosh b)z^{-1}+z^{-2}}$	$\|z\| > \max\{\|e^b\|, \|e^{-b}\|\}$
$\sinh(nb)u[n]$	$\frac{(\sinh b)z^{-1}}{1-(2\cosh b)z^{-1}+z^{-2}}$	$\|z\| > \max\{\|e^b\|, \|e^{-b}\|\}$
$\begin{cases} a^n, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\|z\| > 0$