TRIGONOMETRIC IDENTITIES

$$\cos(u) = \frac{e^{ju} + e^{-ju}}{2}$$

$$\sin(u) = \frac{e^{ju} - e^{-ju}}{2j}$$

$$\cos^2(u) + \sin^2(u) = 1$$

$$\cos^2(u) - \sin^2(u) = \cos(2u)$$

$$2\sin(u)\cos(u) = \sin(2u)$$

$$\cos(u)\cos(v) = \frac{1}{2}\cos(u-v) + \frac{1}{2}\cos(u+v)$$

$$\sin(u)\cos(v) = \frac{1}{2}\sin(u-v) + \frac{1}{2}\sin(u+v)$$

$$\sin(u)\sin(v) = \frac{1}{2}\cos(u-v) - \frac{1}{2}\cos(u+v)$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos^2(u) = \frac{1}{2} + \frac{1}{2}\cos(2u)$$

$$\sin^2(u) = \frac{1}{2} - \frac{1}{2}\cos(2u)$$

Table 2.1 Fourier Series for Several Periodic Signals

Signal (one period)	Coefficients for exponential Fourier series
1. Asymmetrical pulse train; period = T_0: $x(t) = A\Pi\left(\frac{t-t_0}{\tau}\right), \tau < T_0$ $x(t) = x(t+T_0)$, all t	$X_n = \frac{A\tau}{T_0}\operatorname{sinc}(nf_0\tau)\, e^{-j2\pi n f_0 t_0}$ $n = 0, \pm 1, \pm 2, \ldots$
2. Half-rectified sinewave; period = $T_0 = 2\pi/\omega_0$: $x(t) = \begin{cases} A\sin(\omega_0 t), & 0 \le t \le T_0/2 \\ 0, & -T_0/2 \le t \le 0 \end{cases}$ $x(t) = x(t+T_0)$, all t	$X_n = \begin{cases} \frac{A}{\pi(1-n^2)}, & n = 0, \pm 2, \pm 4, \cdots \\ 0, & n = \pm 3, \pm 5, \cdots \\ -\frac{1}{4}jnA, & n = \pm 1 \end{cases}$
3. Full-rectified sinewave; period = $T_0' = \pi/\omega_0$: $x(t) = A\lvert\sin(\omega_0 t)\rvert$	$X_n = \frac{2A}{\pi(1-4n^2)}, n = 0, \pm 1, \pm 2, \ldots$
4. Triangular wave: $x(t) = \begin{cases} -\frac{4A}{T_0}t + A, & 0 \le t \le T_0/2 \\ \frac{4A}{T_0}t + A, & -T_0/2 \le t \le 0 \end{cases}$ $x(t) = x(t+T_0)$, all t	$X_n = \begin{cases} \frac{4A}{\pi^2 n^2}, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$

F.5 FOURIER-TRANSFORM PAIRS

Signal	Fourier transform
$\Pi(t/\tau) = \begin{cases} 1, & \lvert t\rvert \le \tau/2 \\ 0, & \text{otherwise} \end{cases}$	$\tau \operatorname{sinc}(f\tau) = \tau \dfrac{\sin(\pi f \tau)}{\pi f \tau}$
$2W \operatorname{sinc}(2Wt)$	$\Pi(f/2W)$
$\Lambda(t/\tau) = \begin{cases} 1 - \lvert t\rvert/\tau, & \lvert t\rvert \le \tau \\ 0, & \text{otherwise} \end{cases}$	$\tau \operatorname{sinc}^2(f\tau)$
$W \operatorname{sinc}^2(Wt)$	$\Lambda(f/W)$
$\exp(-\alpha t)\, u(t),\ \ \alpha > 0$	$1/(\alpha + j2\pi f)$
$t \exp(-\alpha t)\, u(t),\ \ \alpha > 0$	$1/(\alpha + j2\pi f)^2$
$\exp(-\alpha \lvert t\rvert),\ \ \alpha > 0$	$2\alpha/\left[\alpha^2 + (2\pi f)^2\right]$
$\exp\left[-\pi(t/\tau)^2\right]$	$\tau \exp\left[-\pi(\tau f)^2\right]$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
$u(t)$	$\dfrac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
$1/(\pi t)$	$-j\operatorname{sgn}(f);\ \operatorname{sgn}(f) = \begin{cases} 1, & f > 0 \\ -1, & f < 0 \end{cases}$
$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s);\ \ f_s = 1/T_s$

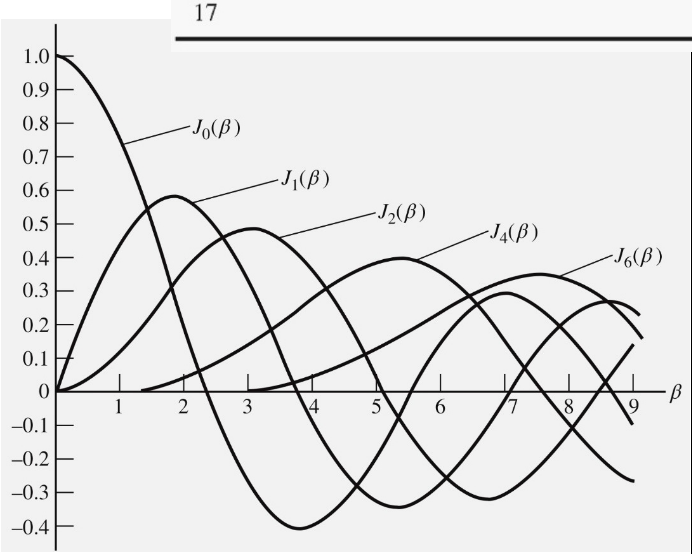
F.6 FOURIER-TRANSFORM THEOREMS

Name	Time-domain operation (signals assumed real)	Frequency-domain operation
Superposition	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(f) + a_2 X_2(f)$
Time delay	$x(t - t_0)$	$X(f) \exp(-j2\pi t_0 f)$
Scale change	$x(at)$	$\lvert a\rvert^{-1} X(f/a)$
Time reversal	$x(-t)$	$X(-f) = X^*(f)$
Duality	$X(t)$	$x(-f)$
Frequency translation	$x(t) \exp(j2\pi f_0 t)$	$X(f - f_0)$
Modulation	$x(t) \cos(2\pi f_0 t)$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
Convolution[4]	$x_1(t) * x_2(t)$	$X_1(f)\, X_2(f)$
Multiplication	$x_1(t)\, x_2(t)$	$X_1(f) * X_2(f)$
Differentiation	$\dfrac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
Integration	$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$X(f)/(j2\pi f) + \frac{1}{2}X(0)\,\delta(f)$

[4] $x_1(t) * x_2(t) \triangleq \int_{-\infty}^{\infty} x_1(\lambda)\, x_2(t - \lambda)\, d\lambda$.

Table 4.1 Values of Selected Bessel Functions

n	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.5$	$\beta = 0.7$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 3.0$	$\beta = 5.0$	$\beta = 7.0$	$\beta = 8.0$	$\beta = 10.0$
0	0.999	0.998	0.990	0.978	0.938	0.881	0.765	0.224	−0.260	−0.178	0.300	0.172	−0.246
1	0.025	0.050	0.100	0.148	0.242	0.329	0.440	0.577	0.339	−0.328	−0.005	0.235	0.043
2		0.001	0.005	0.011	0.031	0.059	0.115	0.353	0.486	0.047	−0.301	−0.113	0.255
3				0.001	0.003	0.007	0.020	0.129	0.309	0.365	−0.168	−0.291	0.058
4						0.001	0.002	0.034	0.132	0.391	0.158	−0.105	−0.220
5								0.007	0.043	0.261	0.348	0.186	−0.234
6								0.001	0.011	0.131	0.339	0.338	−0.014
7									0.003	0.053	0.234	0.321	0.217
8										0.018	0.128	0.223	0.318
9										0.006	0.059	0.126	0.292
10										0.001	0.024	0.061	0.207
11											0.008	0.026	0.123
12											0.003	0.010	0.063
13											0.001	0.003	0.029
14												0.001	0.012
15													0.005
16													0.002
17													0.001



n		β_{n0}	β_{n1}	β_{n2}
0	$J_0(\beta) = 0$	2.4048	5.5201	8.6537
1	$J_1(\beta) = 0$	0.0000	3.8317	7.0156
2	$J_2(\beta) = 0$	0.0000	5.1356	8.4172
4	$J_4(\beta) = 0$	0.0000	7.5883	–
6	$J_6(\beta) = 0$	0.0000	–	–