Exploring Symbolic sums

ECE 4650/5650 Fall 2025

1 Get Things Set Up

1.1 Set the Path to the Signal Processing Package *.mac Files

On Windows XPS 13 tablet:

```
→ /* push("C:\Users\mwickert\Documents\wxmaxima\###.mac", file_search_maxima)$
```

On Linux (WSL) XPS 15 laptop:

```
→ /* push("/home/mwickert/wxmaxima/###.mac", file_search_maxima)$
```

On Linux (WSL) XPS 13 tablet PC:

```
→ /* push("/home/mwickert@xps13/wxmaxima/###.mac", file_search_maxima)$
```

On macOS MacBook Pro:

```
(%i1) push("/Users/markwickert/Documents/Projects/maxima/###.mac", file_search_maxima)$
```

Load the DSPTools Module for DSP Related Plots and Calculations

```
(%i2) load("DSPTools.mac");
(%o2) DSPTools.mac
```

This may require adjustments; units w x h in pixels

```
(%i3) wxplot_size: [600, 450];
wxplot_size [600, 450]
```

2 Solve a Problem

Play with some symbolic summation operations. First learn how to make and forget assumptions.

```
→ assume(abs(a) < 1); /* forget(abs(a) < 1); /* remove an assumption */
(%o4) [|a| < 1]
```

2.1 Try Some Textbook Summations

```
→ ev(sum(a^n,n,0,n),simpsum=true);
```

$$(\%o5)\quad \frac{a^{n+1}-1}{a-1}$$

```
→ ev(sum(a^n,n,0,inf),simpsum=true);
```

$$(\%o6)\quad \frac{1}{1-a}$$

```
→ ev(sum(b^n,n,0,inf),simpsum=true);
```

Is $|b|-1$ positive, negative or zero?

```
→ ev(sum((k),k,0,n),simpsum=true);
```

$$(\%o4)\quad \frac{n^2+n}{2}$$

```
→ factor(ev(sum((k),k,0,n),simpsum=true));
```

$$(\%o19)\quad \frac{n\,(n+1)}{2}$$

2.2 Get to a Problem of Interest

2.2.1 A Solution Interval of the Problem 1b Convolution

First consider just summing over k from 0 to n:

```
S1:ev(sum(k,k,0,n),simpsum=true);
```

(%o9) $\frac{n^2+n}{2}$

Try some additional simplifying such factor():

```
S1_rs:factor(S1);
```

(%o14) $\frac{n\,(n+1)}{2}$

Create a list of numerical values as n runs from 0 to 10:

```
y_out:makelist(subst([n=m],S1),m,0,10);
```

y_out [0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55]

Getting closer to what we really want is summing (N-k) as k runs from 0 to n.

```
(%i4) S2:ev(sum((N-k),k,0,n),simpsum=true);
```

S2 $N\,(n+1)-\frac{n^2+n}{2}$

```
(%i5) S2_rs:factor(S2);
```

S2_rs $-\left(\frac{(n+1)\,(n-2\,N)}{2}\right)$

The above is looking nice for writing out one of the piecewise solution intervals.

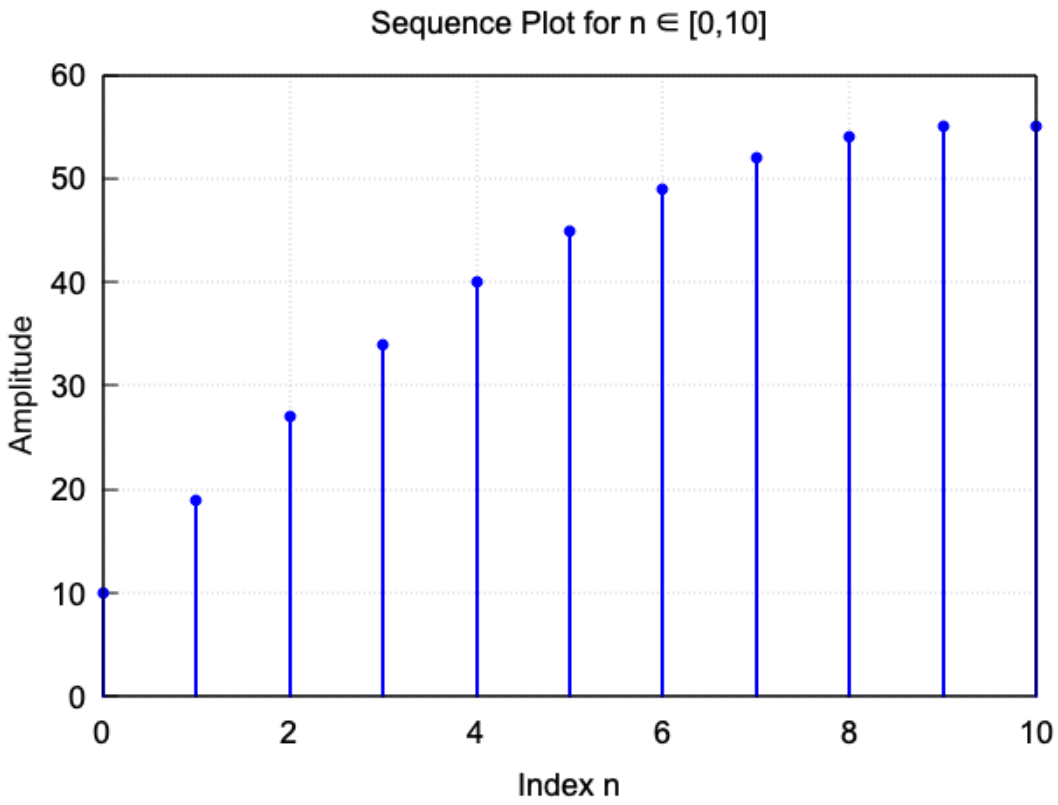
2.2.2 Evaluate S2_rs for case N = 10 and make a list of values as n∈[0,10]

```
(%i6) y_out:makelist(subst([N=10,n=m],S2),m,0,10);
```

y_out [10, 19, 27, 34, 40, 45, 49, 52, 54, 55, 55]

```
(%i7) stemlist(y_out,[],[],"Sequence Plot for n ∈ [0,10]",[],0);
```

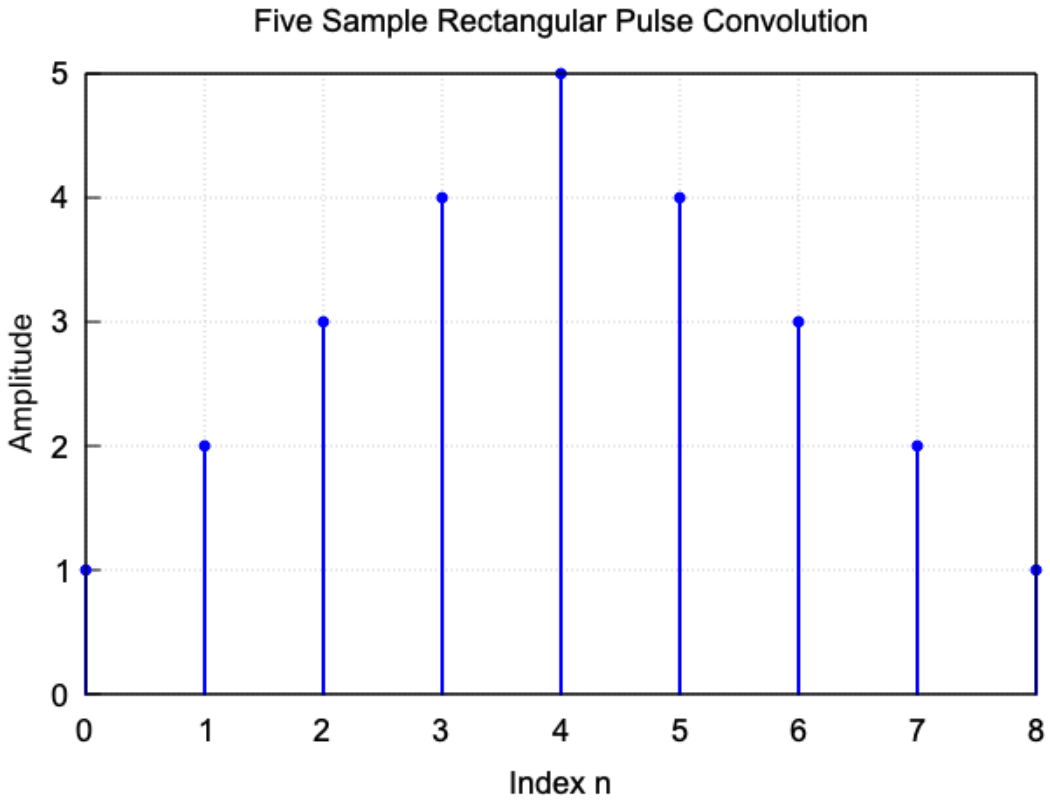
(%t7)



(%o7)

2.3 Direct Convolution of Sequences

```
(%i8)  rect_conv:listconv(ones(5),ones(5));
rect_conv [1,2,3,4,5,4,3,2,1]
(%i9)  stemlist(rect_conv,[],[],"Five Sample Rectangular Pulse Convolution",[],0);
(%t9)
```



(%o9)

2.4 Plotting a Piecewise Solution

Suppose we have a piecewise sequence p1=n/5 for 0≤n≤5 and p2=2 - (n/5)² for 5<n≤10 and zero otherwise. We can contruct a list for plotting that covers n∈[-2,12] as follows:

(%i10)
```
p1:n/5;
```

p1 $\frac{n}{5}$

(%i11)
```
p2:2 - (n/5)^2;
```

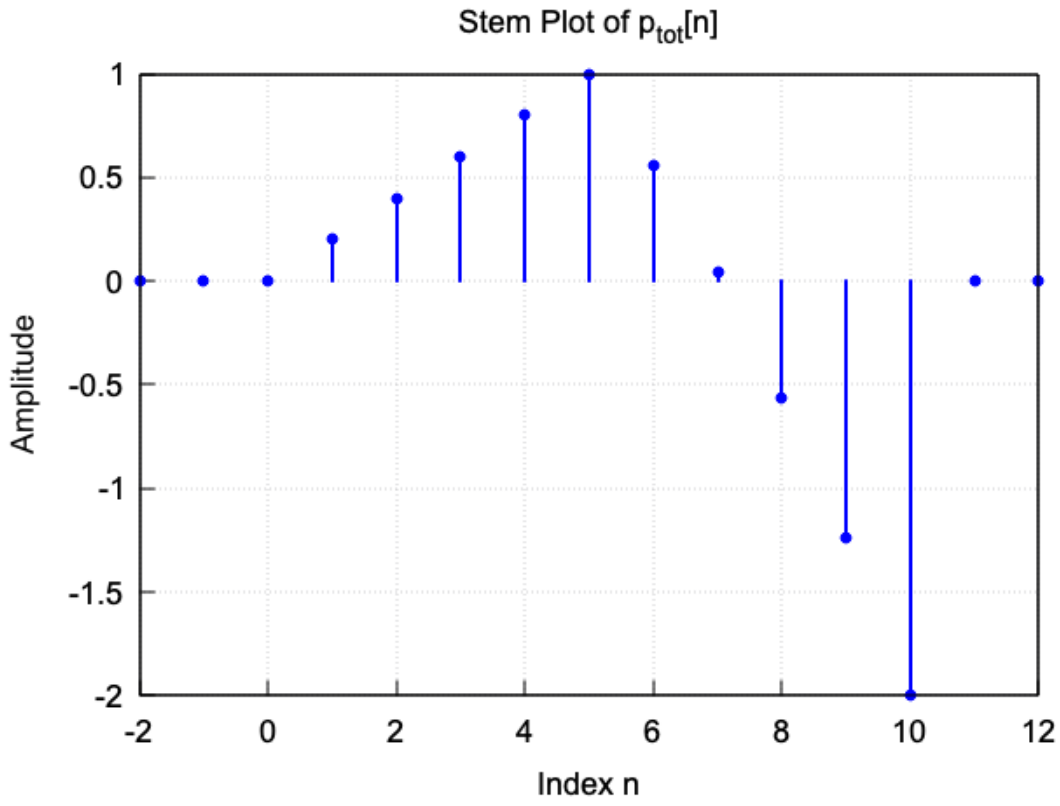
p2 $2 - \frac{n^2}{25}$

(%i12)
```
p_tot:makelist(if m<0 then 0 elseif m<=5 then subst([n=m],p1) elseif m<=10 then subst([n=m],p2) else 0,m,-2,12);
```

p_tot $\left[0,0,0,\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5},1,\frac{14}{25},\frac{1}{25},-\left(\frac{14}{25}\right),-\left(\frac{31}{25}\right),-2,0,0\right]$

(%i13)
```
stemlist(p_tot,[],[],"Stem Plot of p_{tot}[n]",[],-2);
```

(%t13)


Stem Plot of $p_{tot}[n]$
Amplitude
Index n
1
0.5
0
-0.5
-1
-1.5
-2
-2
0
2
4
6
8
10
12

(%o13)